Regenesis and quantum traversable wormholes

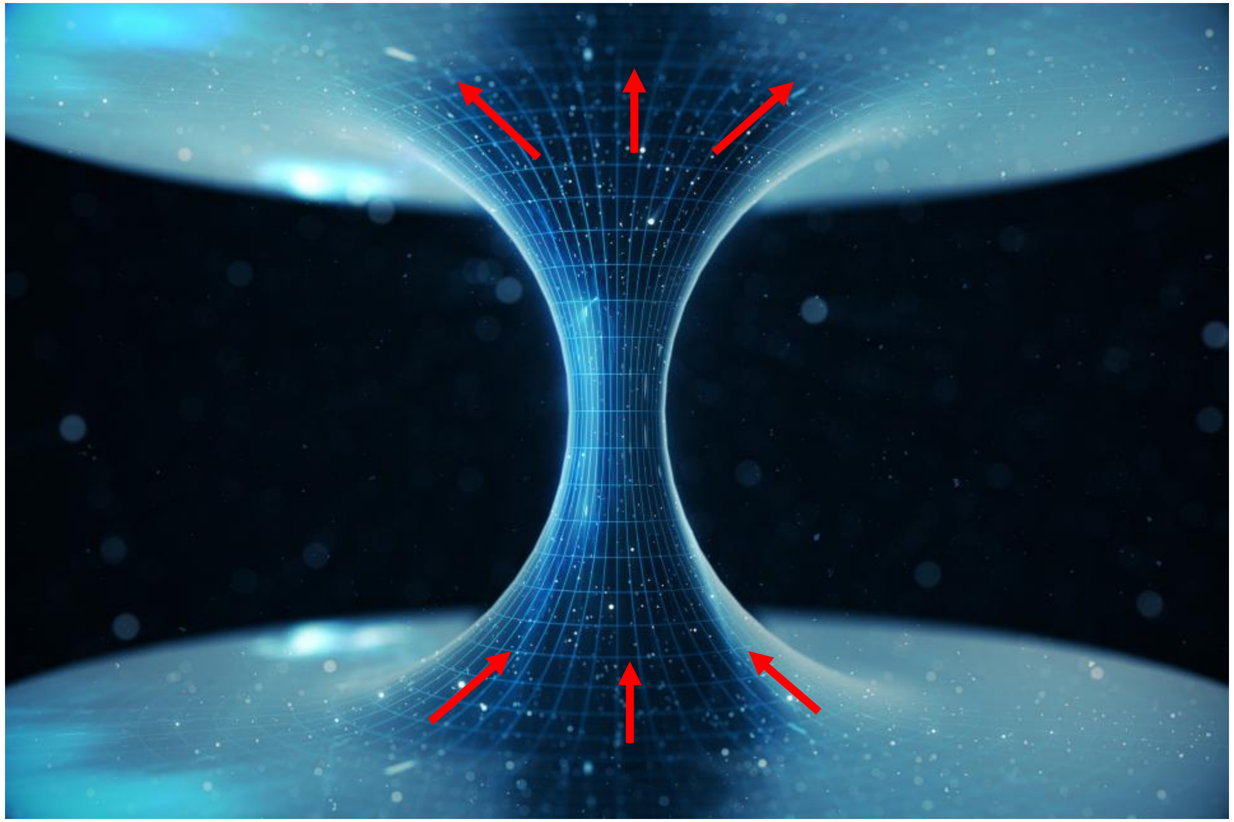
Ping Gao, Harvard University

Outline

1. General introduction
2. Review of gravity side calculation of traversable wormholes
3. A dual CFT calculation: regenesis
4. Outlook

Introduction

What is a wormhole?



Defoucs
Repulsive force
Negative mass

Along null geodesics

$$\int T_{\mu\nu} U^{\mu} U^{\nu} d\lambda < 0$$

Violation of Averaged Null Energy Condition (ANEC)

Foucs
Attractive force
Positive mass

Traversable wormhole is hard

1. Requires matter that violates averaged null energy condition (ANEC).
2. Believed **impossible classically**. e.g. ideal fluid, stress tensor is given by

$$T_{\mu\nu} = (p+\rho)k_\mu k_\nu + p g_{\mu\nu} \implies T_{\mu\nu}U^\mu U^\nu = (p+\rho)(U\cdot k)^2 \geq 0$$

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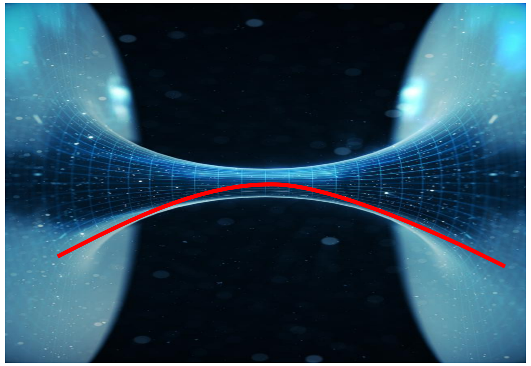
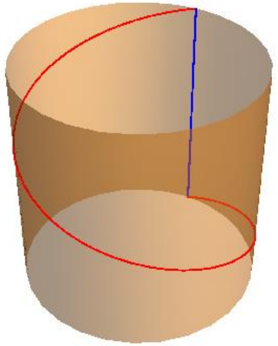
3. Quantum effect is possible to violate ANEC, e.g. Casimir effect.
4. No-go theorem: If the null geodesic is **achronal**, there are strong arguments that the ANEC is satisfied in **QFT**. For example, Generalized Second Law implies ANEC for achronal case.

[Wall]

achronal

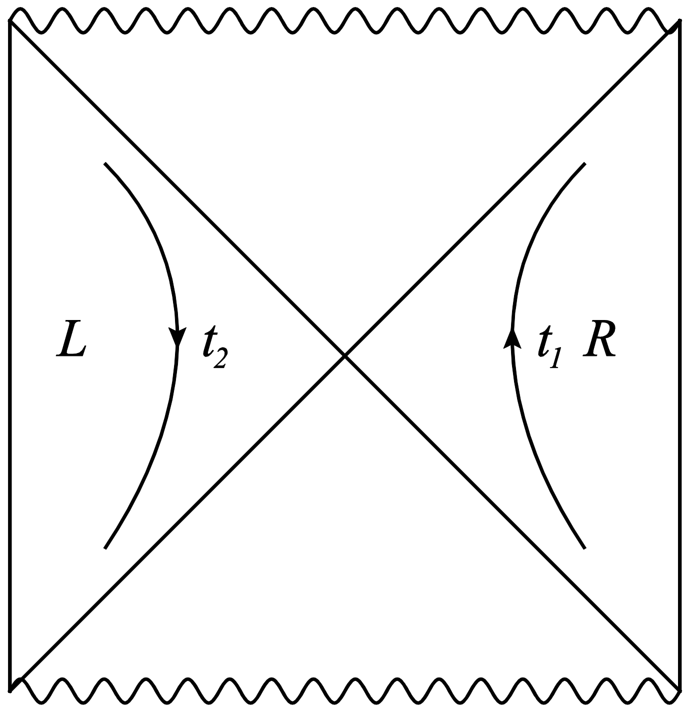


chronal



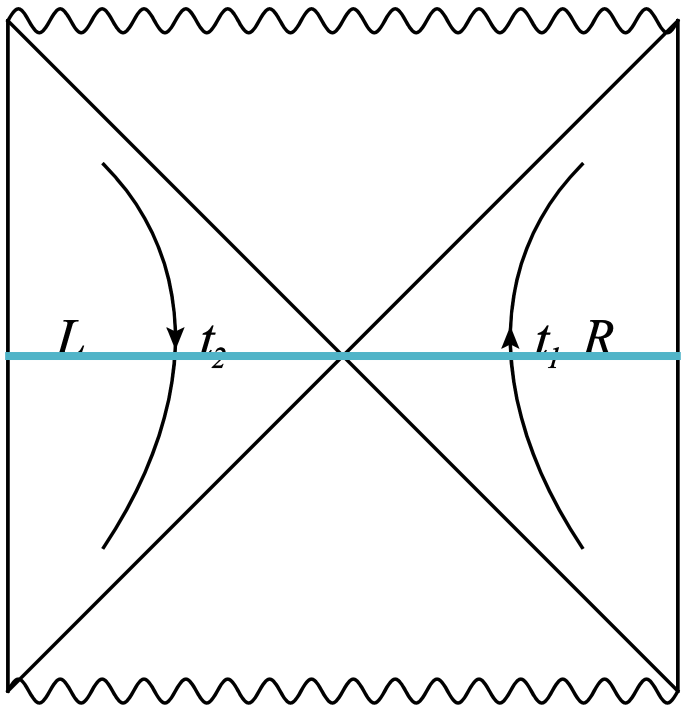
Gravity picture

Eternal AdS-Schwarzschild black hole



1. One CFT on each side, dual to L and R wedge respectively.
2. Hilbert space is product space
$$\mathcal{H} = \mathcal{H}_L \times \mathcal{H}_R$$
3. Decoupled Hamiltonian
$$H = H_L + H_R$$
4. ER bridge, critically non-traversable wormhole due to the existence of Killing symmetry $H_R - H_L$

Eternal AdS-Schwarzschild black hole



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5. t=0 slice, thermofield double state:
$$|\text{tfd}\rangle = \sum_E e^{-\beta E/2} |E, E\rangle$$
[Maldacena 01]
6. ER=EPR

[Maldacena, Susskind]

Couple the two CFT's

1. Start in the eternal black hole state in the decoupled system and turn on an interaction at some time.

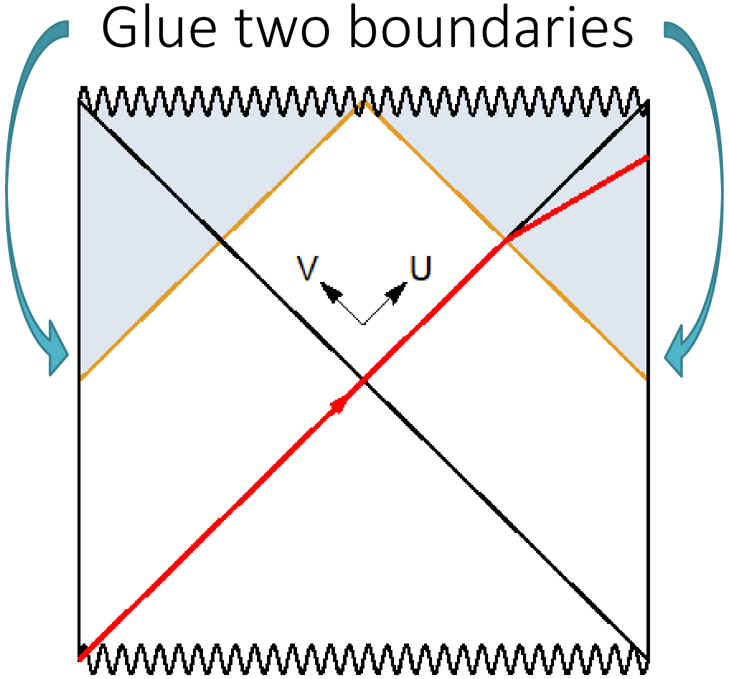
$$\delta S = \int dt\ d^{d-1}x\ h(t,x)\mathcal{O}_R(t,x)\mathcal{O}_L(-t,x),$$

2. This only changes the configuration in the future.
3. Linear order in h, $\langle T_{UU}\rangle$ is proportional to h. Adjust the sign of h to make the averaged null energy negative.

$$\langle T_{UU}\rangle \propto h \implies \int dU\, \langle T_{UU}\rangle < 0 \text{ with one sign of } h$$

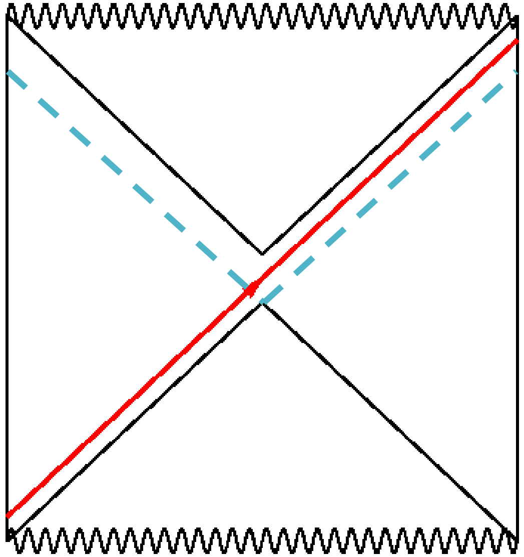
4. Solving linear Einstein equation and null geodesic equation near horizon $V = 0$, we find the red line $V(U)$

$$V(U) = C_0 \int_{-\infty}^{U} dU T_{UU},\ C_0 > 0 \qquad g_{\mu\nu} = g_{0\mu\nu} + h_{\mu\nu}$$



Discussions

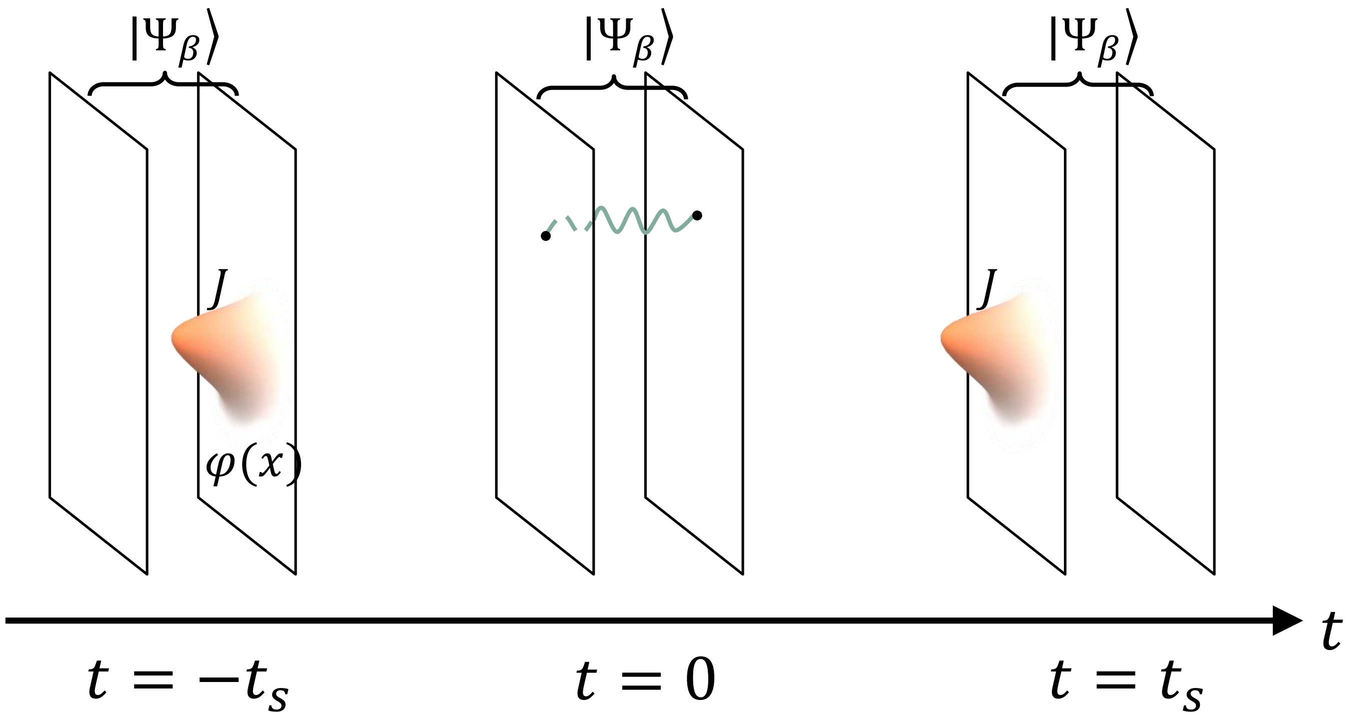
1. Gluing two boundaries is crucial for 1) interaction being local; 2) chronal spacetime avoiding no-go theorem.
2. Width of wormhole is finite, order h. Send signal early.
3. Bulk high energy scattering backreaction. Higher order in h. It has been shown in AdS_2, this gives restriction on total number of particles sent through.
   [Maldacena, Stanford, Yang]
4. Dual to many-body teleportation?
5. Verify ER=EPR. Black hole interiors.



Boundary picture

A CFT picture

In CFT side, we should expect the following phenomenon.



1. Take $|\Psi_\beta\rangle = |TFD\rangle$
2. excitation of signal on one side dissipates.
3. turn on coupling
4. signal reappears from the other side (regenesis)

Add source in the past

1. Change Hamiltonian by adding the source φ^R around $t = -t_s$. $f(t,x)$ is supported around $t = 0$.

$$H = H_0 + H'_S(t) = H_0 + H_S^{(1)}(t) + H_S^{(2)}(t)$$

$$H_S^{(1)} = -g\int d^3\vec{x} f(t,\vec{x})\mathcal{O}^L(\vec{x})\mathcal{O}^R(\vec{x}), \quad H_S^{(2)} = -\int d^3\vec{x}\,\varphi^R(t,\vec{x})J^R(\vec{x})$$

2. Calculate expectation value $\langle J^L(t)\rangle$ at later time $t > 0$.
3. In leading order of source φ^R (linear response). For $f(t,x) = \delta(t)f(x)$.

$$\langle J^L(t)\rangle = i\int ds[e^{-igV}J^L(t)e^{igV}, J^R(s)]\varphi^R(s), \qquad V = \int dx f(x)\mathcal{O}^L(x)\mathcal{O}^R(x)$$

4. $g = 0$, left and right operators commute. No signal.

Scrambling time

1. For the linear response,

$$\langle J^L(t)\rangle = i\int ds[e^{-igV}J^L(t)e^{igV}, J^R(s)]\varphi^R(s), \qquad V = \int dx f(x)\mathcal{O}^L(x)\mathcal{O}^R(x)$$

   if t is small, as J^L and $\mathcal{O}$ are independent few-body operators, $[J^L(t), V] \approx 0$. No signal. Similarly if t_s is small, we can move J^R across V and get no signal.

2. We define the time scale that this commutator is order 1 as scrambling time t_*.

3. We can also define the commutator as causal propagator from right to left. This relates to a bulk interpretation.

$$\langle J^L(t)\rangle = \int ds G^{LR}(t,s)\varphi^R(s), \qquad G^{LR}(t,t') \equiv i\theta(t-t')(W(t,x') - W^*(t,t'))$$

$$W(t,t') = \langle \Psi_\beta | e^{-igV}J^L(t)e^{igV}J^R(t')|\Psi_\beta\rangle$$

Entanglement properties of TFD

Let us review some entanglement properties of TFD $|\Psi_\beta\rangle = Z_\beta^{-1/2} \sum_E e^{-\beta E/2} |E, E\rangle$

1. For one side observer, it behaves like thermal density matrix.

$$\langle \Psi_\beta | \mathcal{O}_1^L \cdots \mathcal{O}_n^L | \Psi_\beta \rangle = \frac{1}{Z_\beta} \mathrm{Tr}\left(e^{-\beta H} \mathcal{O}_1 \cdots \mathcal{O}_n \right) \equiv \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_\beta$$

2. Thermal dissipation for one side correlation function with large time and space separation.

$$G^{RR}(t, \vec{x}, t', \vec{x}') \sim e^{-\frac{|t-t'|}{\tau_r}}, \qquad G^{RR}(t, \vec{x}, t', \vec{x}') \sim e^{-\frac{|\vec{x}-\vec{x}'|}{\ell_r}}$$

3. KMS feature: all operators can be written as one side operators

$$J^R |\Psi_\beta\rangle = J^L(i\beta/2) |\Psi_\beta\rangle \implies J^R(t) |\Psi_\beta\rangle = J^L(-t + i\beta/2) |\Psi_\beta\rangle$$

4. Left-right correlation supports around $t = -t_s$: $\langle \Psi_\beta | J^L(t) J^R(-t_s) | \Psi_\beta \rangle \sim e^{-|t-t_s|/\tau_r}$

Late times

Let us focus on a simple case: $t, t_s \gg t_*$ and assume $f(x) = \delta(x)$.

$$W(t,-t_s) = \sum_n \frac{(ig)^n}{n!} \langle \Psi_\beta | e^{-igV} J^L(t) (\mathcal{O}^L(0)\mathcal{O}^R(0))^n J^R(-t_s) | \Psi_\beta \rangle$$

$$= \sum_n \frac{(ig)^n}{n!} \langle \Psi_\beta | e^{-igV} \underbrace{J^L(t) (\mathcal{O}^L(0))^n J^L(t_s + i\beta/2) (\mathcal{O}^L(i\beta/2))^n}_{} | \Psi_\beta \rangle$$

Out of time ordered. Vanish for late times in chaotic system.

[Maldacena, Shenker, Stanford]

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Out of time ordered. Vanish for late times in chaotic system.

$$\sim \langle \Psi_\beta | e^{-igV} J^L(t) J^L(t_s + i\beta/2) | \Psi_\beta \rangle$$

$$= \sum_n \frac{(-ig)^n}{n!} \langle \Psi_\beta | \underbrace{(\mathcal{O}^L(-i\beta/2))^n (\mathcal{O}^L(0))^n J^L(t) J^L(t_s + i\beta/2)}_{} | \Psi_\beta \rangle$$

time ordered. Factorize for late times in chaotic system.

$$\sim \langle e^{-igV} \rangle \langle \Psi_\beta | J^L(t) J^L(t_s + i\beta/2) | \Psi_\beta \rangle$$

[Maldacena, Shenker, Stanford]

Late times

At late times, the structure is simple.

$$W(t,-t_s) \sim \langle e^{-igV} \rangle \langle \Psi_\beta | J^L(t) J^L(t_s + i\beta/2) | \Psi_\beta \rangle$$

1. The expectation $\langle e^{-igV} \rangle$ is generally complex number. Regenesis happens.

$$\langle J^L(t) \rangle = \int ds G^{LR}(t,s) \varphi^R(s), \qquad G^{LR}(t,t') \equiv i\theta(t-t')(W(t,x') - W^*(t,t'))$$

2. W is proportional to $\langle \Psi_\beta | J^L(t) J^R(-t_s) | \Psi_\beta \rangle = \langle \Psi_\beta | J^L(t) J^L(t_s + i\beta/2) | \Psi_\beta \rangle$ only supported around $t = -t_s$. Regenesis at the symmetric time for a short while.

3. If we have large species of $\mathcal{O}$ operators or integration over large spatial region

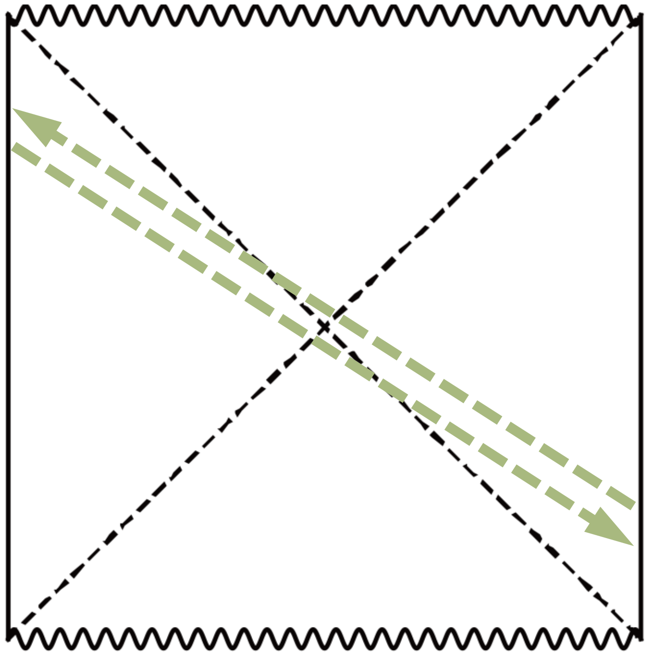
$$V = \frac{1}{K} \sum_{i=1}^{K} \mathcal{O}_i^L(0) \mathcal{O}_i^R(0) \text{ or } V = \frac{1}{A(\mathcal{D})} \int_{\mathcal{D}} d\vec{x} \mathcal{O}_i^L(\vec{x}) \mathcal{O}_i^R(\vec{x}) \implies \boxed{\langle e^{-igV} \rangle \sim e^{-ig\langle V \rangle}} + O(1/K, 1/A(\mathcal{D}))$$

Pure phase

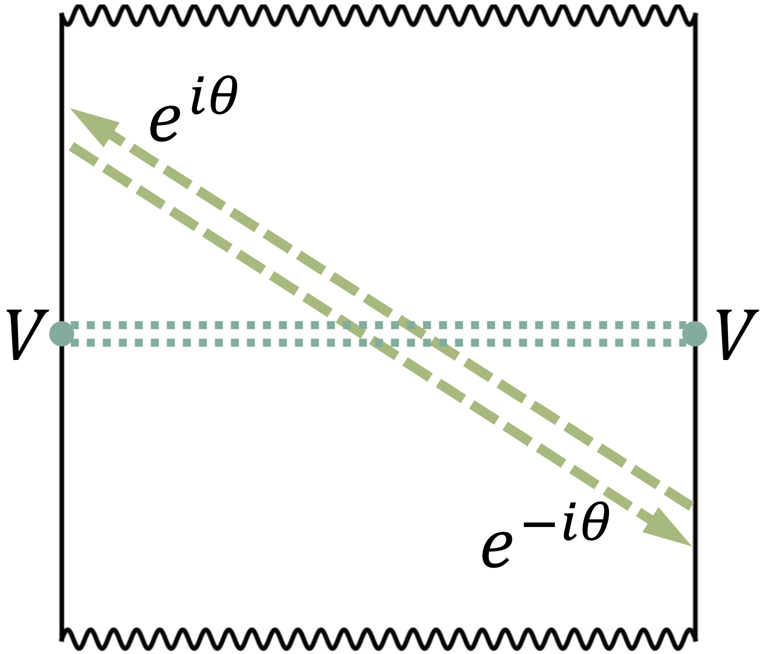
Interference interpretation

At late times, regenesis has an interference interpretation that is **not semi-classical**.

$$G^{LR}(t, -t_s) = i\left(e^{-ig\langle V\rangle}\left\langle J^L(t)J^R(-t_s)\right\rangle - h.c.\right)$$



$\Longrightarrow$



Signal is quantum in nature

To determine a signal is classical or quantum, compare its expectation value with fluctuation in thermodynamical limit. Take spin system as an example. J measures the average spin $J = \frac{1}{N}\sum \sigma_i^z$.

In large N limit, we expect (U_L is the unitary adding excitation by a source)

$$\tilde{J} = \left\langle \Psi_\beta | U_L^\dagger J^L U_L | \Psi_\beta \right\rangle \sim O(1),\ \delta\tilde{J} = \left[\left\langle \Psi_\beta | U_L^\dagger (J^L - \tilde{J})^2 U_L | \Psi_\beta \right\rangle\right]^{1/2} \sim N^{-1/2}$$

One the other hand, one can show for our setup **in late times**

$$\begin{aligned} \bar{J}_g &\equiv \langle J^L(t) \rangle \approx a \langle J^L U^R \rangle + h.c. \\ &\leq 2|a|(\langle J^L J^L \rangle)^{1/2} = 2|a| J_2 \\ (\delta J_g)^2 &\equiv \left\langle (J^L(t) - \bar{J}_g(t))^2 \right\rangle_g \\ &= (1 - 2\,\mathrm{Re}\, a) J_2^2 - \bar{J}_g^2 + b \end{aligned}$$

$$U^R = e^{i\int ds\, \varphi^R(s) J^R(s)}, \qquad a = \left\langle e^{-igV} \right\rangle_\beta - 1$$

$$J_2 = \sqrt{\langle J^2 \rangle_\beta} \qquad J_4 = \left(\langle J^4 \rangle_\beta\right)^{\frac{1}{4}}$$

$$b \equiv a \langle \Psi_\beta | (J^L(t))^2 U^R | \Psi_\beta \rangle + h.c. \leq 2|a| J_4^2$$

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→

$$\bar{J}_g \sim \delta J_g \sim J_2 \sim N^{-1/2}$$

NOT survive in classical limit!

2D CFT calculation

Late time regime is universal for chaotic system. The regime $t \sim t_*$ is not universal and depends on the details of theory. Here we sketch calculation of 2D CFT in the limit $c \gg h_J \gg h_\mathcal{O} \sim O(1)$ limit.

Using BCH formula and KMS feature to write all right operators as left ones

$$W(t,x;-t_s,x_s) = \sum_{n=0}^{\infty} \frac{(-ig)^n}{L^n n!} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(\prod_{k=1}^{n} dx_k \right) W_n$$

$$W_n = \left\langle [\mathcal{O}_n, [\mathcal{O}_{n-1}, \cdots [\mathcal{O}_1, J] \cdots] \, \tilde{J} \tilde{\mathcal{O}}_n \cdots \tilde{\mathcal{O}}_1 \right\rangle_\beta$$

$$J \equiv J(t,x), \qquad \mathcal{O}_i \equiv \mathcal{O}(0,x_i), \qquad \tilde{\mathcal{O}}_i \equiv \mathcal{O}(i\beta/2, x_i), \qquad \tilde{J} \equiv J(t_s + i\beta/2, x_s)$$

To calculate each W_n is to calculate a multi-pt function with two heavy J operators.

2D CFT calculation

For $w_n = \langle J(t_a, x_a) J(t_b, x_b) \mathcal{O}(t_1, x_1) \cdots \mathcal{O}(t_{2n}, x_{2n}) \rangle_\beta$ evaluated in thermal ensemble, we first do a conformal transformation (imaginary time has period of β)

$$z = e^{\frac{2\pi}{\beta}(x+t)}, \qquad \bar{z} = e^{\frac{2\pi}{\beta}(x-t)}$$

As J is heavy, we can first do a special conformal transformation according to its weight to introduce a branch cut between two J's. Then the metric becomes curved, and we can treat w_n as 2n point function of $\mathcal{O}$'s in this curved background.

This special conformal transformation automatically take the leading order contribution of identity Virasoro block between J's and $\mathcal{O}$'s into account.

$$1 - w = \left(1 - \frac{z_{ab} z}{z_b(z_a - z)}\right)^\alpha, \quad \alpha = \sqrt{1 - 24\frac{h_J}{c}}$$

Maps z_a to $w_a = \infty$, and z_b to w_b =1
In w plane, the branch cut is from 1 to infinity.

[Fitzpatrick, Kaplan, Walters, Wang]

2D CFT calculation

For example, 4 pt function, with $1-w=\left(1-\frac{z_{ab}z}{z_b(z_a-z)}\right)^{\alpha}$, $\alpha=\sqrt{1-24\frac{h_J}{c}}$

$$
\begin{aligned}
\langle J_a J_b \mathcal{O}_1 \mathcal{O}_2\rangle_z &= \mathbb{J}_a^{-h_J}\mathbb{J}_b^{-h_J}\mathbb{J}_1^{-h_{\mathcal{O}}}\mathbb{J}_2^{-h_{\mathcal{O}}}\langle J_a J_b \mathcal{O}_1 \mathcal{O}_2\rangle_w \\
&\approx \mathbb{J}_a^{-h_J}\mathbb{J}_b^{-h_J}\mathbb{J}_1^{-h_{\mathcal{O}}}\mathbb{J}_2^{-h_{\mathcal{O}}}\langle J_a J_b\rangle_w \langle 0_w|\mathcal{O}_1 \mathcal{O}_2\rangle_w \\
&= \langle J_a J_b\rangle_z \mathbb{J}_1^{-h_{\mathcal{O}}}\mathbb{J}_2^{-h_{\mathcal{O}}}\frac{1}{w_{12}^{2h_{\mathcal{O}}}}
\end{aligned}
$$

Using cross ratio u: $\mathcal{V}(u)\equiv\frac{\langle J_a J_b \mathcal{O}_1 \mathcal{O}_2\rangle_z}{\langle J_a J_b\rangle_z\langle \mathcal{O}_1 \mathcal{O}_2\rangle_z}=\left(\frac{z_{12}^2}{\mathbb{J}_1\mathbb{J}_2 w_{12}^2}\right)^{h_{\mathcal{O}}}=\left(\frac{\alpha^2u^2(1-u)^{\alpha-1}}{(1-(1-u)^{\alpha})^2}\right)^{h_{\mathcal{O}}}$, $\quad u=\frac{z_{12}z_{ab}}{z_{1a}z_{2b}}$

In large t case, u approaches to zero but on first sheet for time ordered case, and on second sheet for out of time ordered case.

This illustrates the different behaviors of different time orderings.

[Roberts, Stanford]

2D CFT calculation

Using same techniques, one can derive in large c limit

$$\frac{\langle J_a J_b \mathcal{O}_1 \cdots \mathcal{O}_{2n}\rangle_z}{\langle J_a J_b\rangle_z} = \frac{\mathbb{J}_a^{-h_J}\mathbb{J}_b^{-h_J}}{\langle J_a J_b\rangle_z}\left(\prod_{i=1}^{2n}\mathbb{J}_i^{-h_\mathcal{O}}\right)\langle J_a J_b \mathcal{O}_1 \cdots \mathcal{O}_{2n}\rangle_w$$

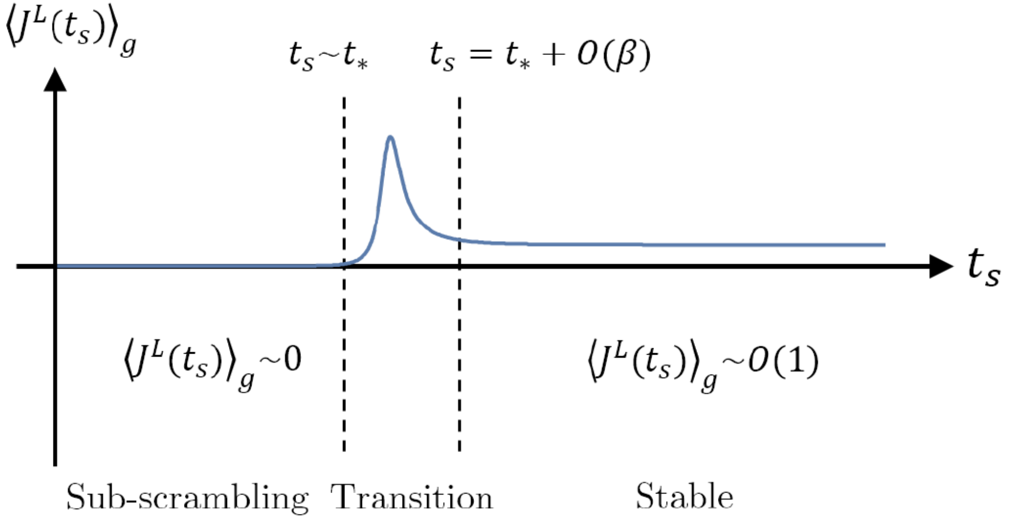
$$= \sum_{\{(s_{2i},s_{2i+1})\}}\prod_{i=1}^{n}\left[\mathcal{V}(u_{s,i})\left\langle \mathcal{O}_{s_{2i}}\mathcal{O}_{s_{2i+1}}\right\rangle_z\right], \qquad u_{s,i} \equiv \frac{z_{s_{2i},s_{2i+1}}z_{ab}}{z_{s_{2i},a}z_{s_{2i+1},b}}$$

In large species or spatially integrated length cases, W becomes simple: replacing $\langle V\rangle$ by a 4-pt function. The exponent takes scattering effect into account.

$$W(t,-t_s) = \exp\left(-ig\frac{\langle[\mathcal{O}^L(0),J^L(t)]O^R(0)J^R(-t_s)\rangle}{\langle J^L(t)J^R(-t_s)\rangle}\right)\langle J^L(t)J^R(-t_s)\rangle$$

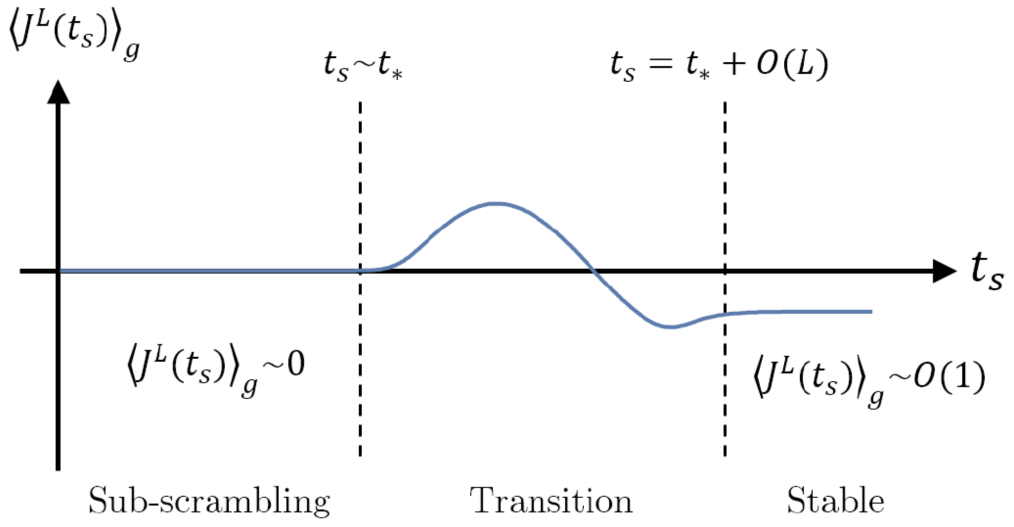
Plot

As the result is also proportional to $\langle J^L(t) J^R(-t_s)\rangle$, regenesis only happens around symmetric time $t \sim t_s$. We plot $\langle J^L(t_s)\rangle$ as a function of t_s.



(a)

Large species



(b)

Large spatial integrated length

Robustness

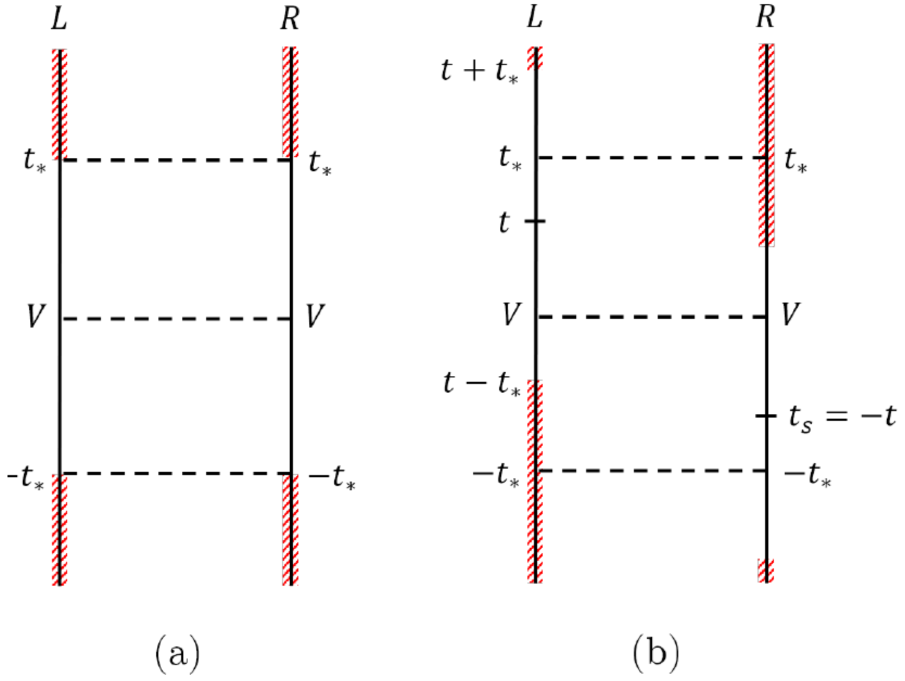
Regenesis requires two features: effective coupling and entanglement.

$$W(t,-t_s) = \exp\left(-ig\frac{\langle[\mathcal{O}^L(0), J^L(t)]O^R(0)J^R(-t_s)\rangle}{\langle J^L(t)J^R(-t_s)\rangle}\right)\langle J^L(t)J^R(-t_s)\rangle$$

One can show that if we change the state from TFD to $\gamma^L(t_0)\,|\Psi_\beta\rangle$ with a perturbation at t_0 from left, regenesis will be violated when

$$|t_0| \gg t_* \quad \text{(destroy the coupling between two systems)}$$

$$|t - t_0| \gg t_* \quad \text{(destroy correlation of } J^L \text{ and } J^R\text{)}$$



Discussion

1. Reverse time ordering. Early signal reappears late.
2. Early stage is semiclassical with backreaction (4-pt function is dual to bulk scattering), late stage is quantum.
3. Requires careful preparation of the thermal field double state (robustness).
4. Late time implies a quite different picture without geometry interpretation: quantum traversable wormhole? A deeper understanding of quantum traversable wormhole is required.
5. This calculation does not contain bulk causal lightcone picture (semiclassical without backreaction in MSY's AdS_2 analysis), which is the case when $h_J \sim h_O \sim 1$ and $g \to \infty$ limit. This actually can be achieved by a different analysis of 2D CFT. That is the case G^{LR} has a lightcone pole when $t \sim t_s \sim t_*$. [Gao, Liu: to appear]

Conclusion and Outlook

1. Similar phenomena in chaotic field theory system of traversable wormholes. Signals reappear from dissipation from the entangled partner: regenesis.
2. Not enough for teleportation. What is the right protocol?
3. Old cats never die: to be or not to be?
4. Traversable wormhole has a feature that particles escaping from horizon. Could this help understanding Hawking radiation in an alternative way? Relation to quantum information paradox and Hayden-Preskill protocol?
5. Probe behind horizon. Help bulk reconstruction in quantum error correction formalism?
6. Experimental realization?

Thank you!

Ping Gao, Harvard University