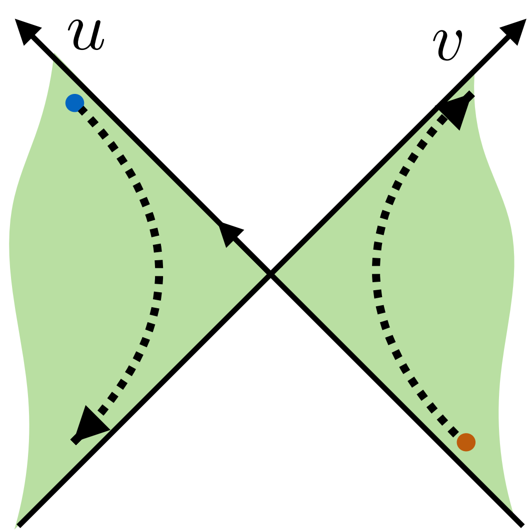
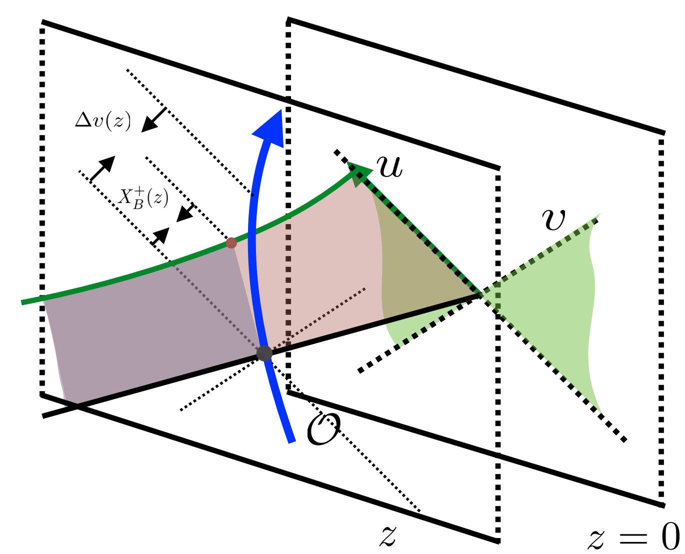
Energy Condition, Modular Flow, and AdS/CFT

Black Holes and Holography Workshop,TSIMF, Jan 7-11, 2019

Huajia Wang

Kavli Institute for Theoretical Physics





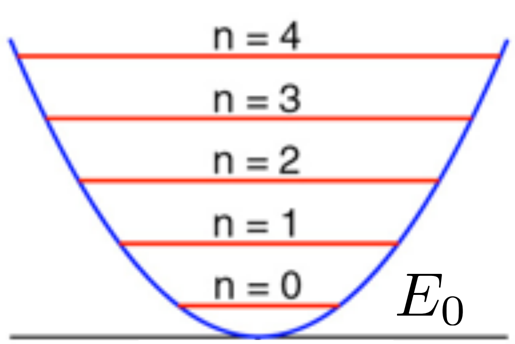
arXiv:1806.10560; arXiv:1706.09432; JHEP 1609 038(2016)

S. Balakrishnan, T. Faulkner, R. Leigh, M. Li, Z. Khandker, O. Parrikar, H. Wang

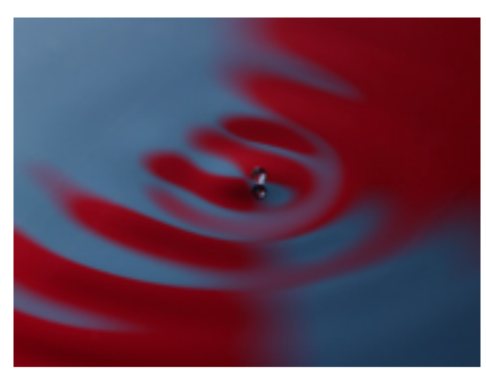
Energy Conditions

What are they?

- unitarity of QM: positivity of total energy
- extended systems (QFT): local energy/momentum density
- constraints on energy/momentum density



$$E \geq 0$$

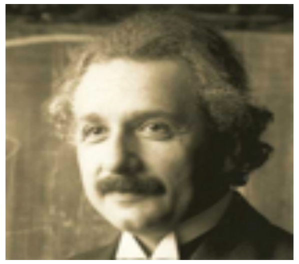


$$E = \int_{\mathcal{R}^n} dx^n \, \mathcal{E}(x) \geq 0$$

Energy Conditions

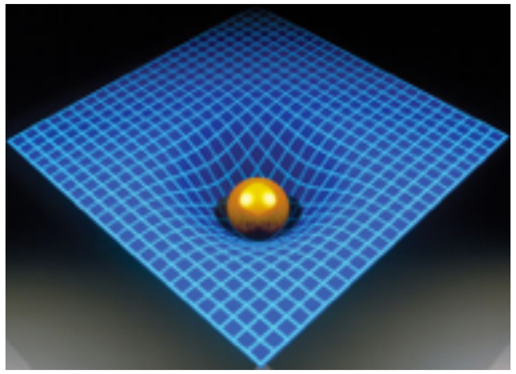
Why do we care?

- classical: important in general relativity
- energy-momentum = spacetime geometry
- energy conditions = constraints on spacetime



Einstein's equations:

$$G_{\mu\nu} = 8\pi\, T_{\mu\nu}$$



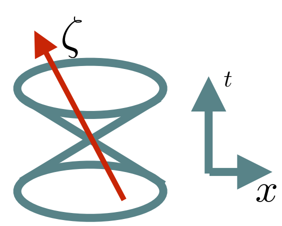
Energy Conditions

<u>Why do we care?</u>

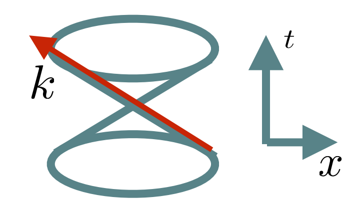
examples: Hawking, Ellis, 1973

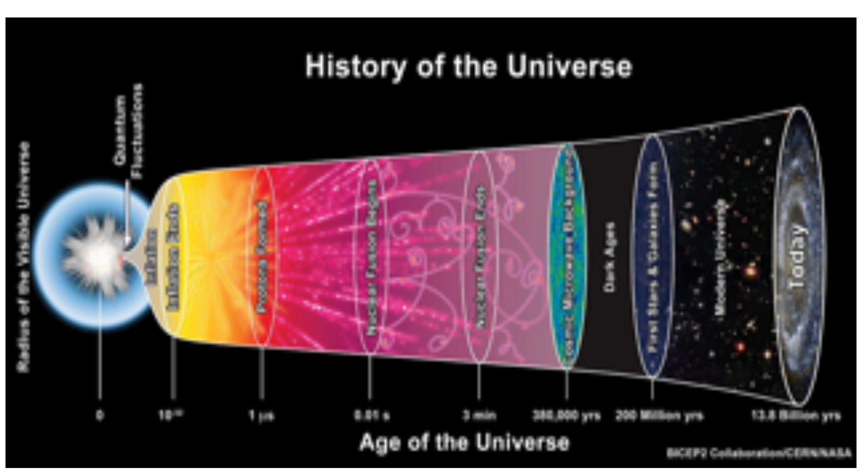
- strong energy condition (SEC) —> singularity theorem
- null energy condition (NEC) —> horizon area theorem

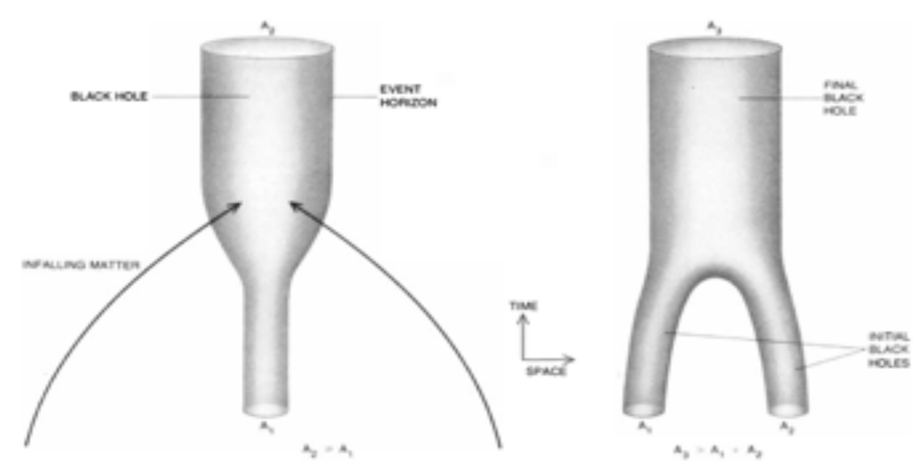
$$\left(T_{ab} - \frac{1}{2} T g_{ab}\right) \zeta^a \zeta^b \geq 0$$



$$T_{ab} \, k^a k^b \geq 0$$







CERTAIN PROPERTIES OF BLACK HOLES suggest that there is a resemblance between the area of the event horizon of a black hole and the concept of entropy in thermodynamics. As matter and radiation continue to fall into a black hole (*space-time configuration at left*) the area of the cross section of the event horizon steadily increases. If two black holes collide and merge (*configuration at right*), the area of the cross section of the event horizon of the resulting black hole is greater than the sum of the areas of the event horizons of the initial black holes. The second law of thermodynamics says that the entropy of an isolated system always increases with passage of time.

Energy Conditions

<u>What do we want?</u>

in QM: QFTs in fixed background spacetime

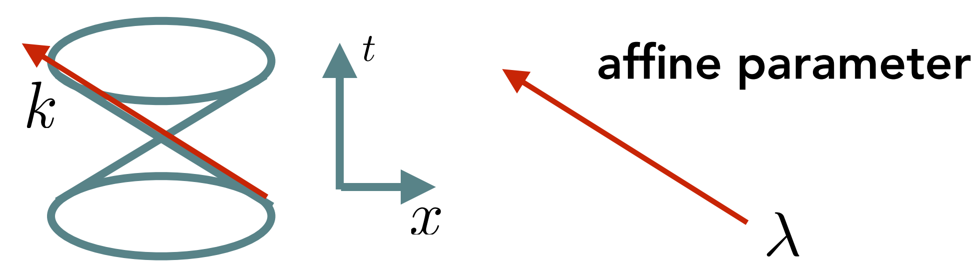
- constraints on $\langle \hat{T}_{\mu\nu} \rangle_\psi$
- NEC violated by quantum effects: e.g. Casimir effect
- correct modification to NEC?
- two main conjectures:

<u>Averaged Null Energy Condition (ANEC)</u>

<u>Quantum Null Energy Condition (QNEC)</u>

AVERAGED NULL ENERGY CONDITION (ANEC)

$$\int d\lambda \, \langle \hat{T}_{\mu\nu} \rangle_\psi k^\mu k^\nu \geq 0$$



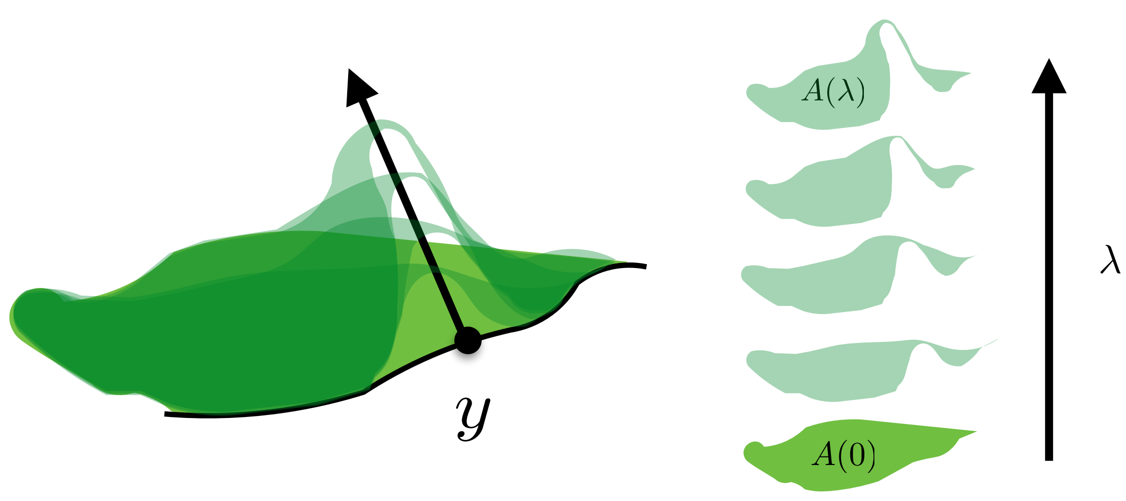
Why? violation leads to causality breakdown

— supports traversable wormhole/time machine

M. Morris, K. Thorne, U. Yurtsever, PRL. 61. 13. 1988

QUANTUM NULL ENERGY CONDITION (QNEC)

$$\langle \hat{T}_{\mu\nu}(y) \rangle_\psi k^\mu k^\nu \geq \partial_\lambda^2 S_{A(\lambda)}(\psi)$$



Motivation: generalized second law

<u>Can we prove them in QFTs? How?</u>

A brief history of proofs…

for specific types of theories:

- ANEC for free scalar and Maxwell fields;

G. Klinkhammer, 1991; L. Ford, T. Roman, 1995; A. Folacci, 1992

- ANEC for 2d massive QFTs;

R. Verch, 2000

- QNEC for free/super-renormalizable fields;

R. Busso, Z. Fisher, J. Koeller, S. Leichenaber, A. Wall, 2015

A brief history of proofs…

for holographic theories (a broad class of CFTs):

- proof of ANEC using AdS/CFT: W. Kelly, A. Wall, 2014

  causality constraint in the bulk.

- proof of QNEC using AdS/CFT: J. Koeller, S. Leichenauer, 2016; C. Akers. V. Chandrasekaran, S. Leichenaber, A. Levin, A. Moghaddam, 2017

  entanglement wedge nesting (EWN)

<u>Can we do better?</u>

<u>Proofs for generic QFT/CFTs?</u>

<u>Can we do better?</u>
<u>Proofs for generic QFT/CFTs?</u>

Recent progresses…

Can we do better?
Proofs for generic QFT/CFTs?

- ANEC in relativistic QFTs: T. Faulkner, R. Leigh, O. Parrikar, H. Wang, 2016

  monotonicity of relative entropy

- ANEC in CFTs: T. Hartman, S. Kundu, A. Tajdini, 2016

  causality of correlation functions in light-cone limit

- QNEC in CFTs: S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

  causality of correlation function under modular flow

Plan of the talk:

- Review of AdS/CFT proofs (ANEC + QNEC)
- Summary of general field theory proofs (ANEC + QNEC)
- Bulk modular flow in AdS/CFT
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Proving ANEC using AdS/CFT

$$\int_{-\infty}^{\infty} dx^+ \langle \hat{T}_{++} \rangle_\psi \geq 0$$

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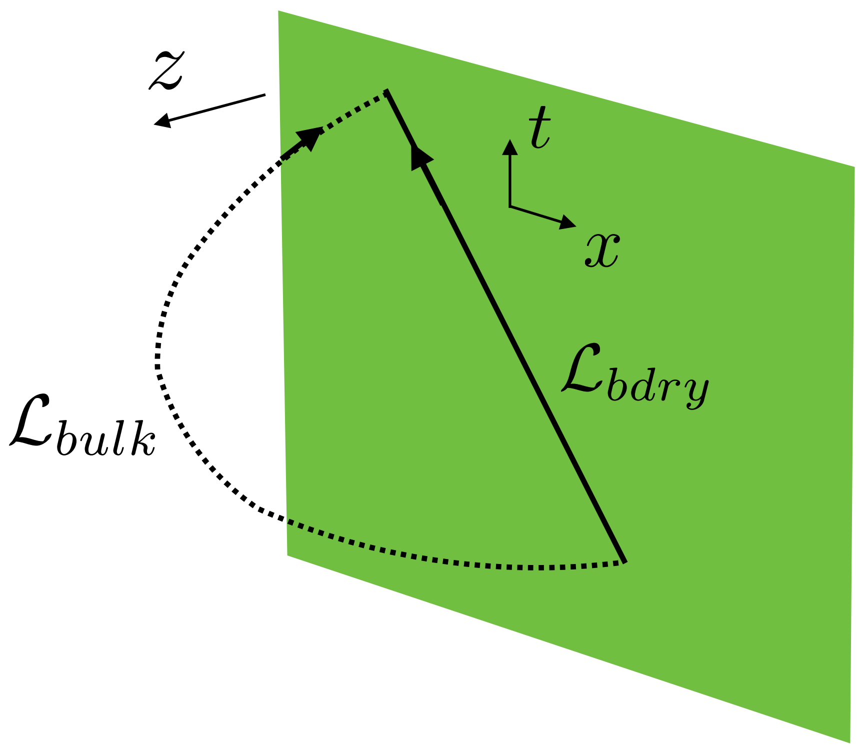

$$\mathcal{L}_{bulk} \geq \mathcal{L}_{bdry}$$

S. Gao, R. Wald, 2000

“bulk respects boundary causality”

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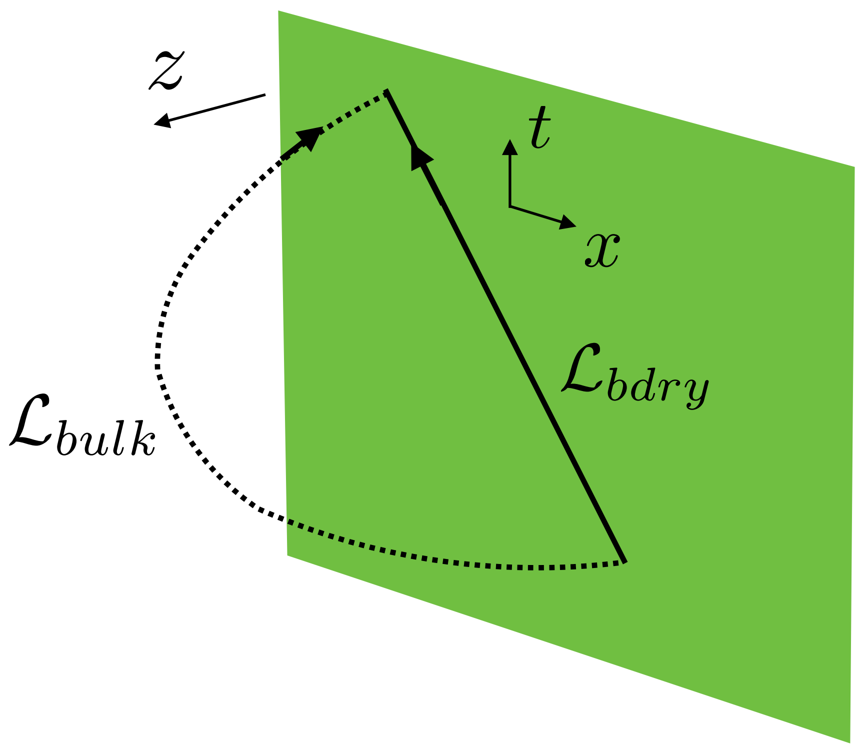

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As a GR result, can be proved by assuming that the "ANEC" in the bulk theory is satisfied

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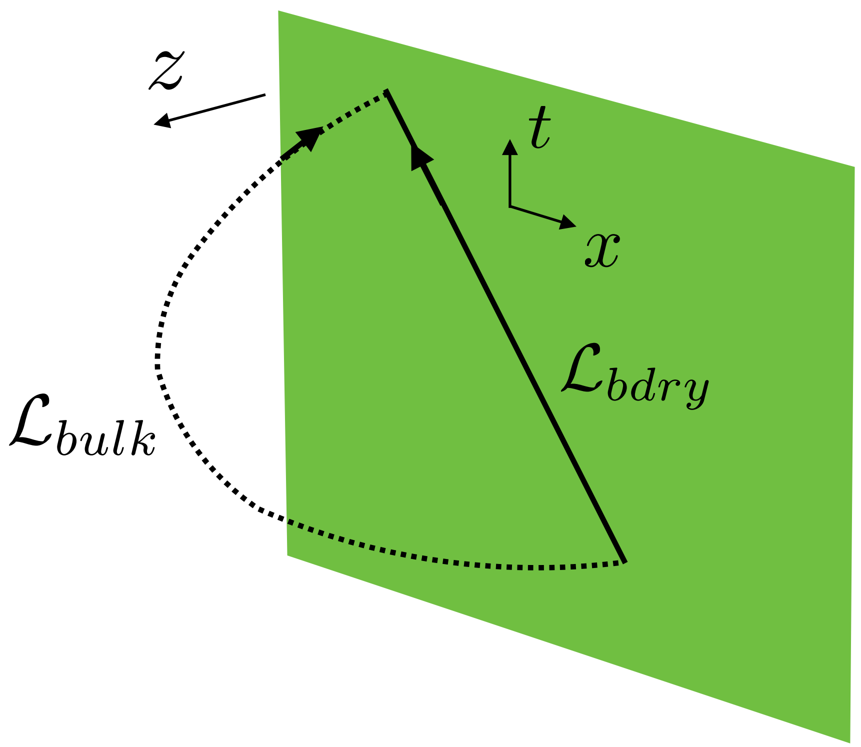

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In AdS/CFT, via Fefferman-Graham gauge expansion:

$$ds^2 = \frac{R^2}{z^2} \left\{ dz^2 + \left[\eta_{ab} + z^d \frac{16\pi G}{dR^{d-1}} \langle T_{ab} \rangle_\psi + \mathcal{O}\left(z^{d+2}\right) \right] dx^a dx^b \right\}, \quad z \to 0$$

Proving ANEC using AdS/CFT

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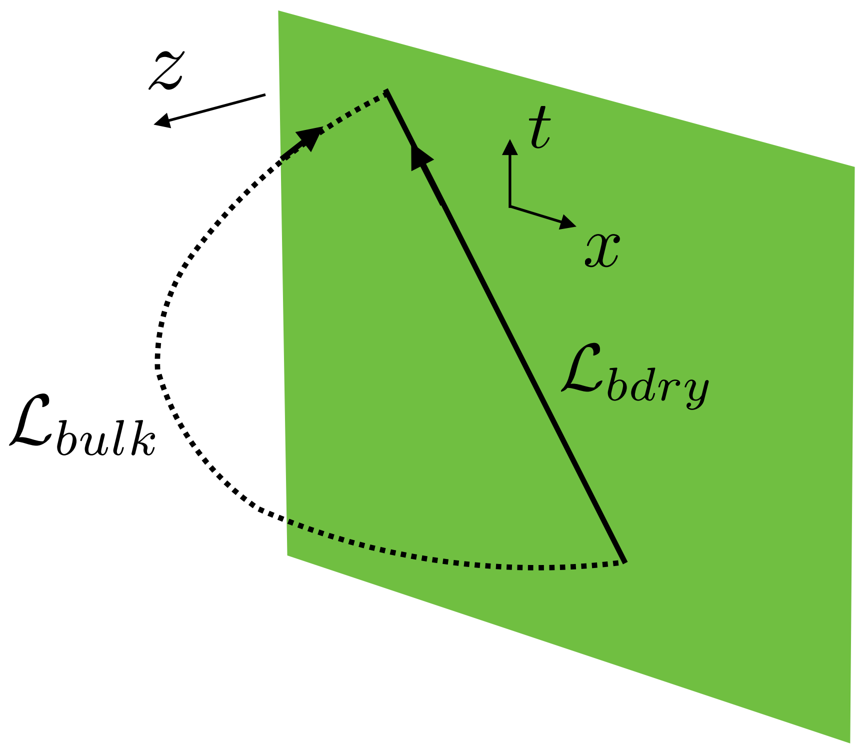

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Gao-Wald's conclusion as consistent condition for holographic CFTs

$\lim z \to 0$ ⇒ $$\int_{-\infty}^{\infty} dx^+ \langle \hat{T}_{++} \rangle_\psi \geq 0$$ boundary ANEC ✓

leading order constraint in F. G. gauge expansion

Proving QNEC using AdS/CFT

$$\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$$

J. Koeller, S. Leichenauer, 2016; C. Akers. V. Chandrasekaran,
S. Leichenaber, A. Levin, A. Moghaddam, 2017

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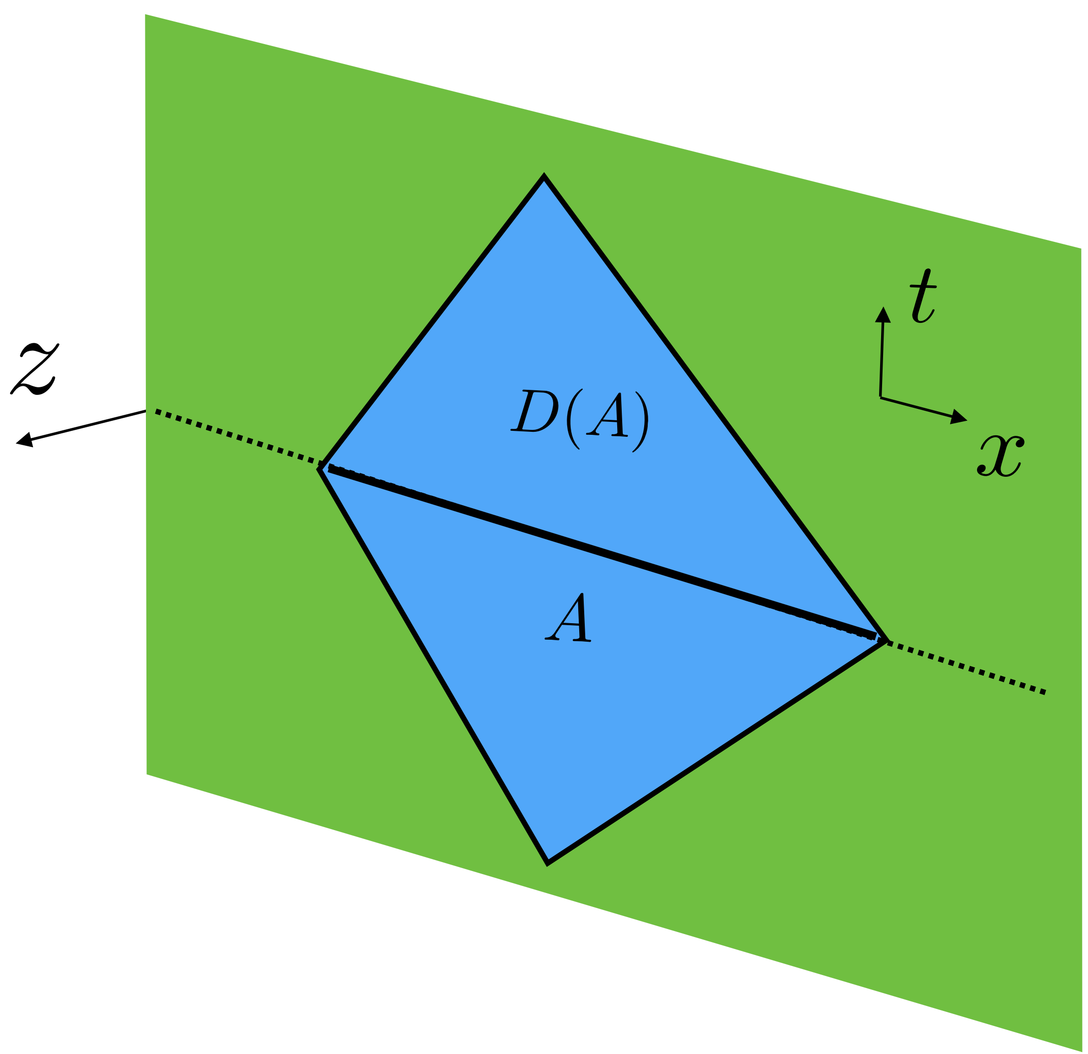
“bulk reconstruction in entanglement wedges”



AdS/CFT: bulk physics can be “reconstructed” from the boundary

how much bulk region can be reconstructed from CFT operators localized in D(A)?

subregion duality

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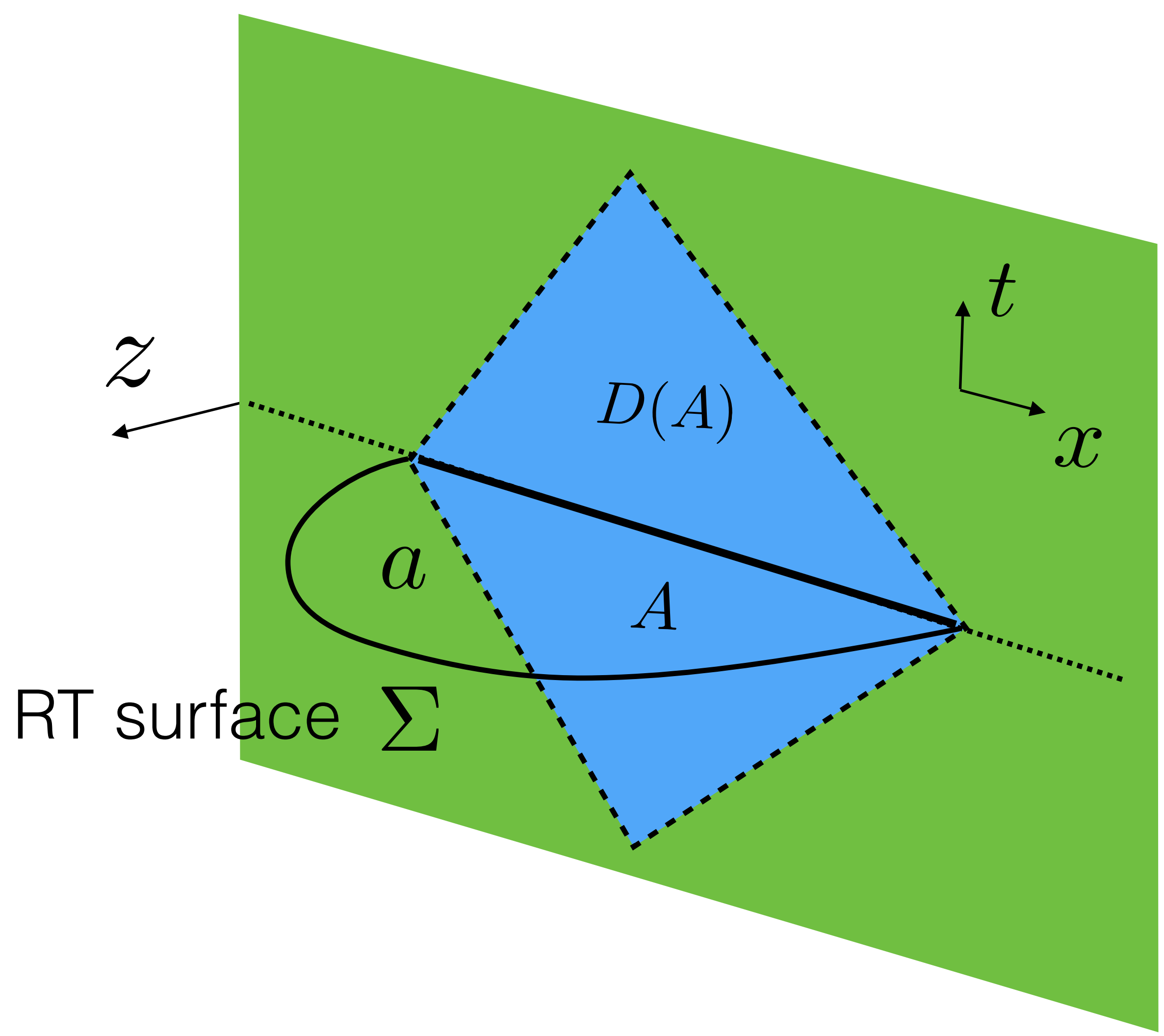

strong evidence:
entanglement wedge

X. Dong, D. Harlow, A. Wall, 2016

$$\partial a = \Sigma \cup A$$

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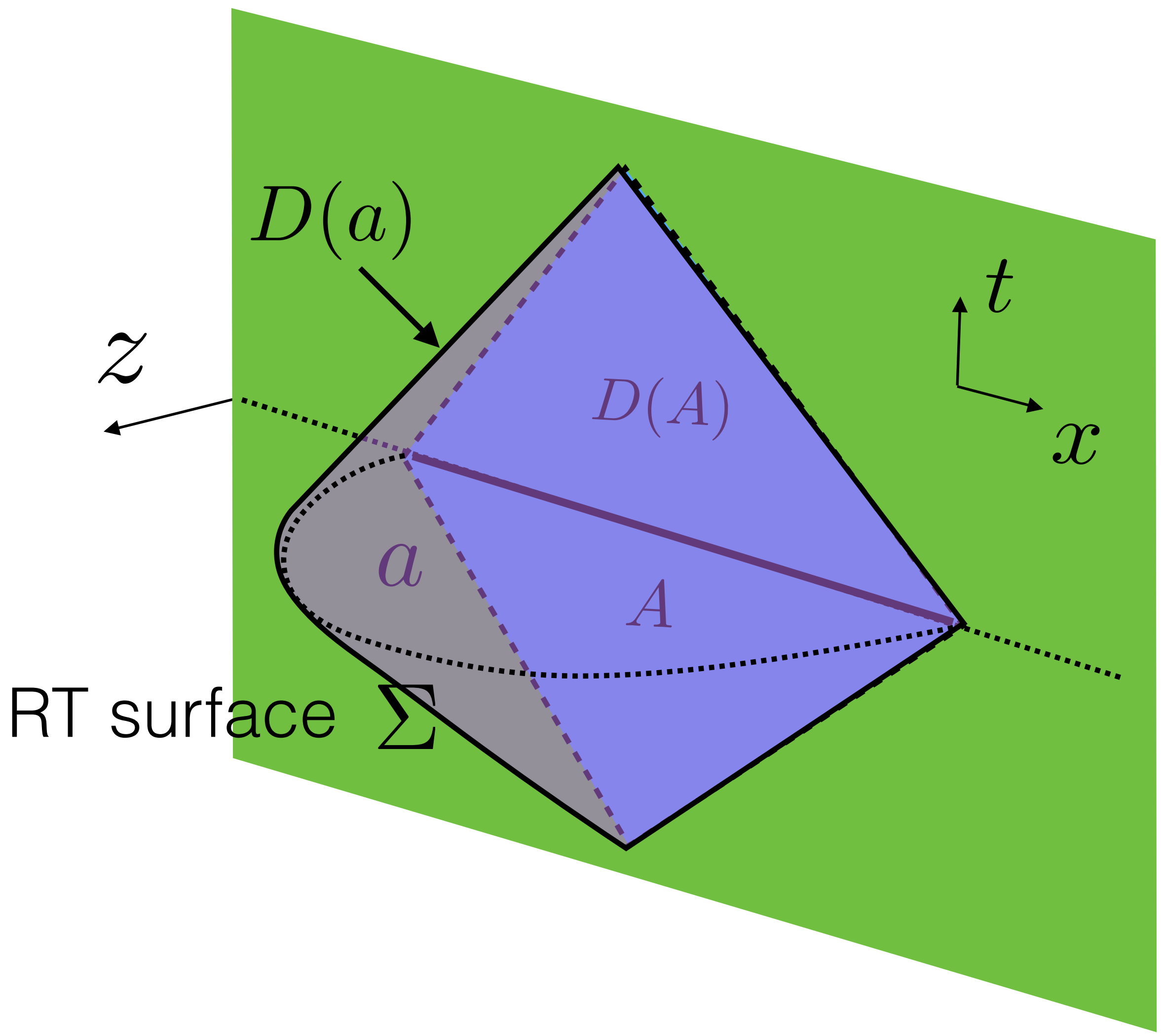

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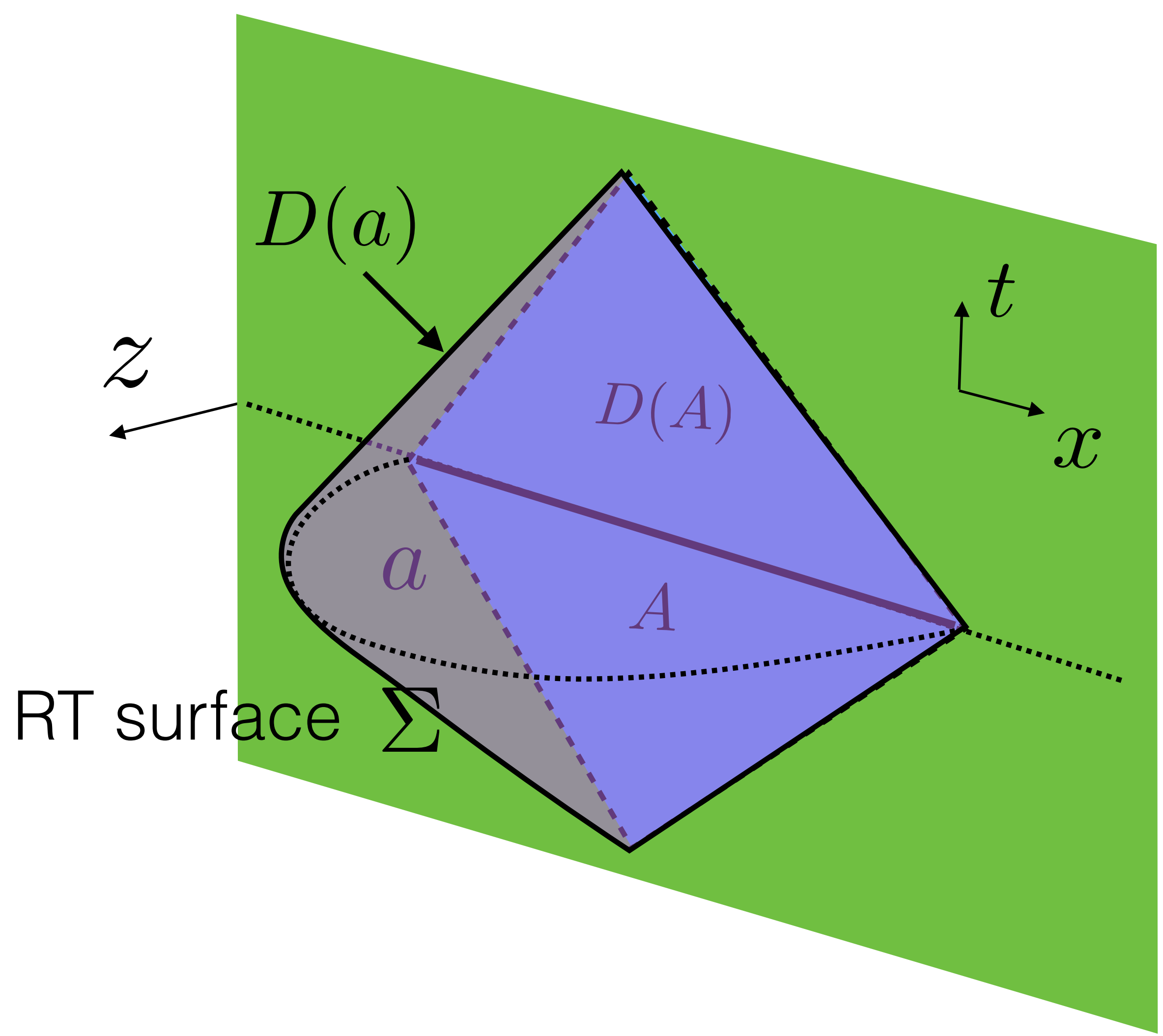

strong evidence:
entanglement wedge

X. Dong, D. Harlow, A. Wall, 2016

$$\partial a = \Sigma \cup A$$

entanglement wedge = D(a)

$$D(a) \text{``} \approx \text{''} D(A)$$

Proving QNEC using AdS/CFT $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

J. Koeller, S. Leichenauer, 2016; C. Akers. V. Chandrasekaran, S. Leichenaber, A. Levin, A. Moghaddam, 2017

Entanglement Wedge Nesting (EWN): $D(\tilde{A}) \subseteq D(A) \to D(\tilde{a}) \subseteq D(a)$

Proving QNEC using AdS/CFT

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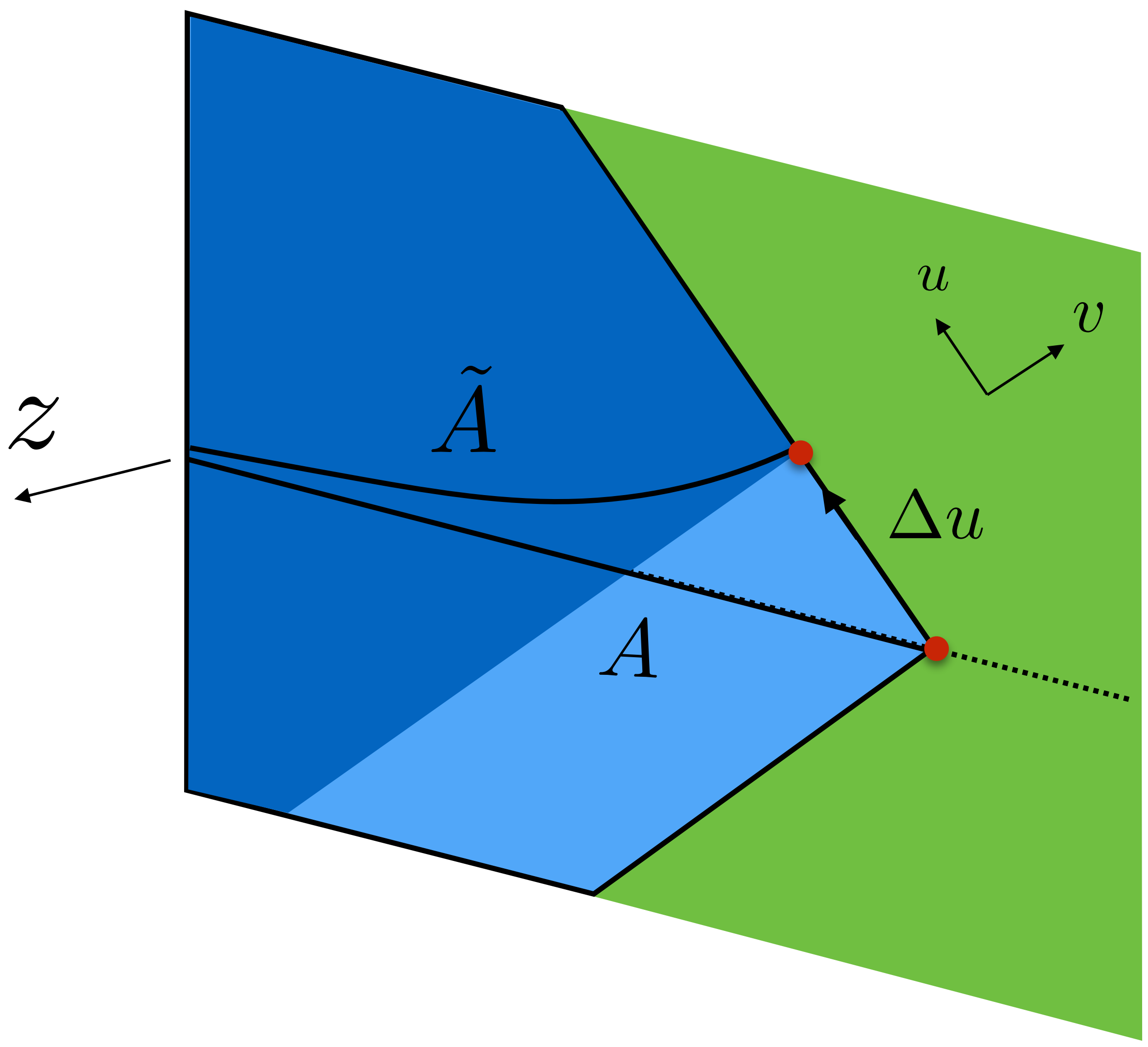

at the boundary:

$\Delta u \geq 0$: null deformation

$D(\tilde{A}) \subseteq D(A)$

Proving QNEC using AdS/CFT

$$\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$$

J. Koeller, S. Leichenauer, 2016; C. Akers. V. Chandrasekaran, S. Leichenaber, A. Levin, A. Moghaddam, 2017

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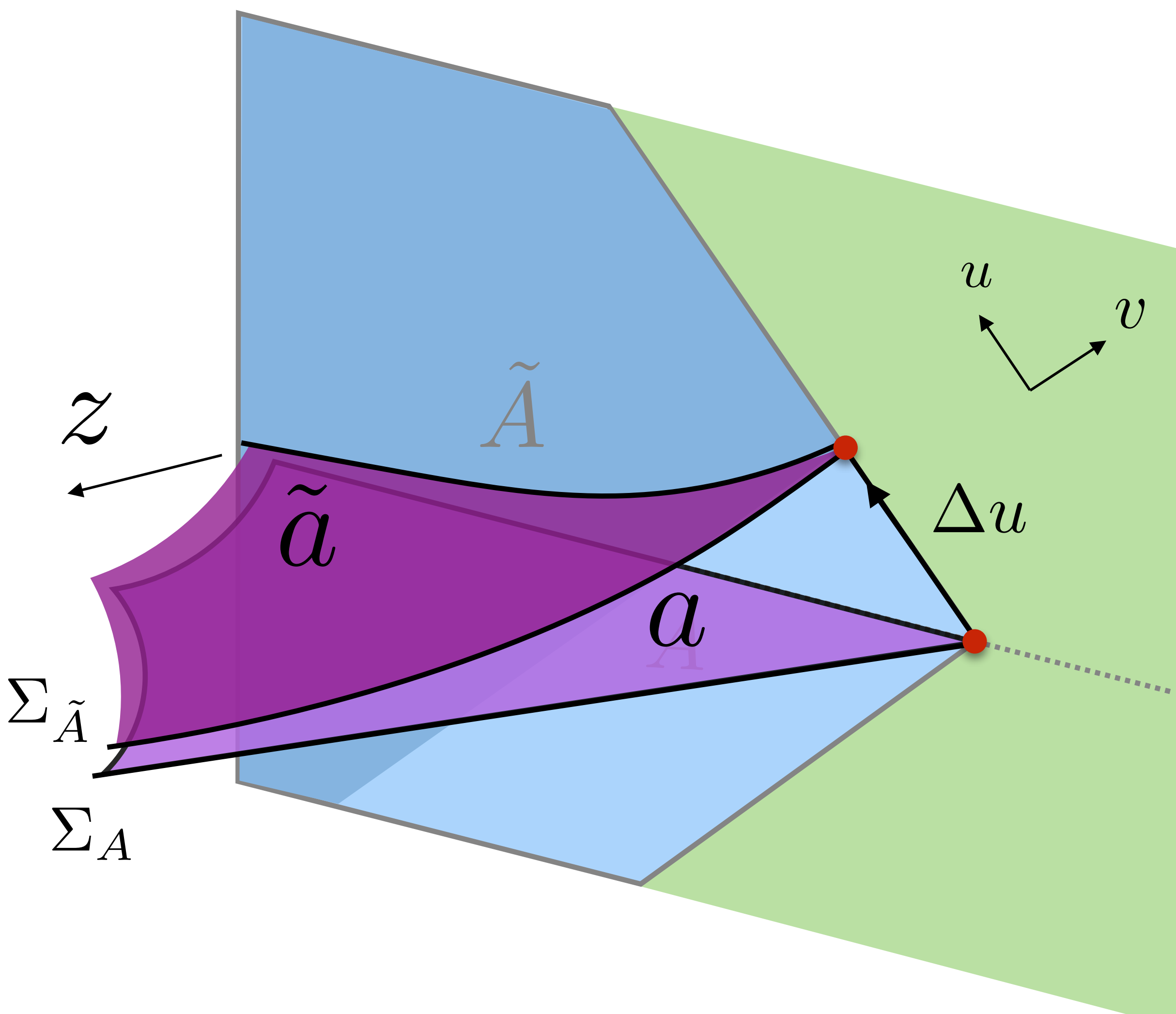

at the boundary:

$\Delta u \geq 0$: null deformation

$D(\tilde{A}) \subseteq D(A)$

into the bulk:

$D(\tilde{a}) \subseteq D(a)$ (EWN)

$\Sigma_{\tilde{A}}$ spacelike/null Σ_A

RT surfaces dynamics

Proving QNEC using AdS/CFT

$$\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$$

J. Koeller, S. Leichenauer, 2016; C. Akers. V. Chandrasekaran, S. Leichenaber, A. Levin, A. Moghaddam, 2017

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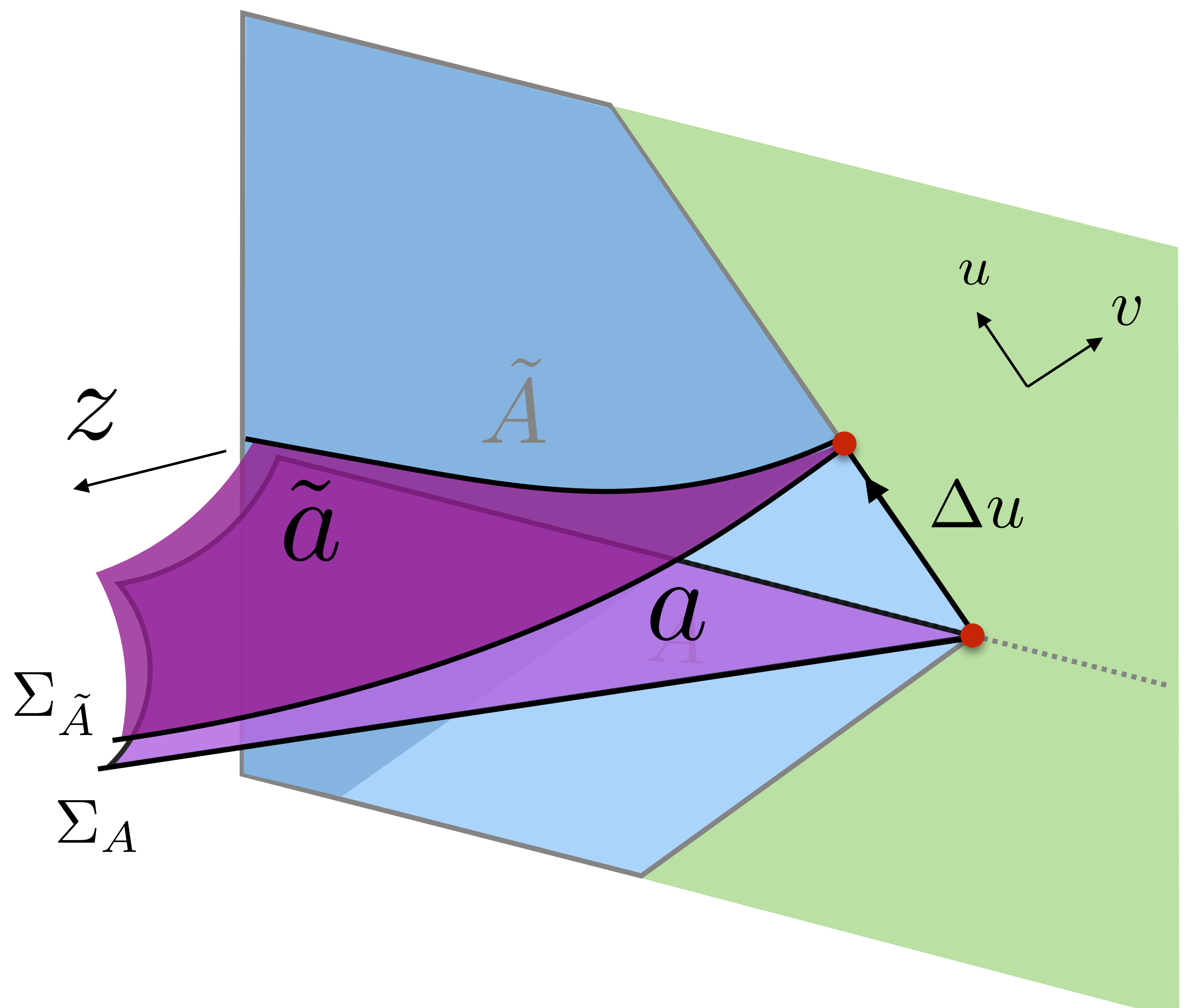

$\Sigma_{\tilde{A}}$ spacelike/null Σ_A

near boundary expansion:

(F-G gauge)

$$g_{uu} = \frac{16\pi G}{dR^{d-3}} z^{d-2} \langle T_{ab}\rangle_\psi + \mathcal{O}(z^d)$$

$$X^i_{\Sigma_A}(z) = X^i_{\partial A} + \frac{4G}{dR^{d-1}} z^d \partial_i S_{EE}(A) + \mathcal{O}(z^{d+1})$$

$$X^i_{\Sigma_{\tilde{A}}}(z) = X^i_{\partial \tilde{A}} + \frac{4G}{dR^{d-1}} z^d \partial_i S_{EE}(\tilde{A}) + \mathcal{O}(z^{d+1})$$

Proving QNEC using AdS/CFT

$$\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$$

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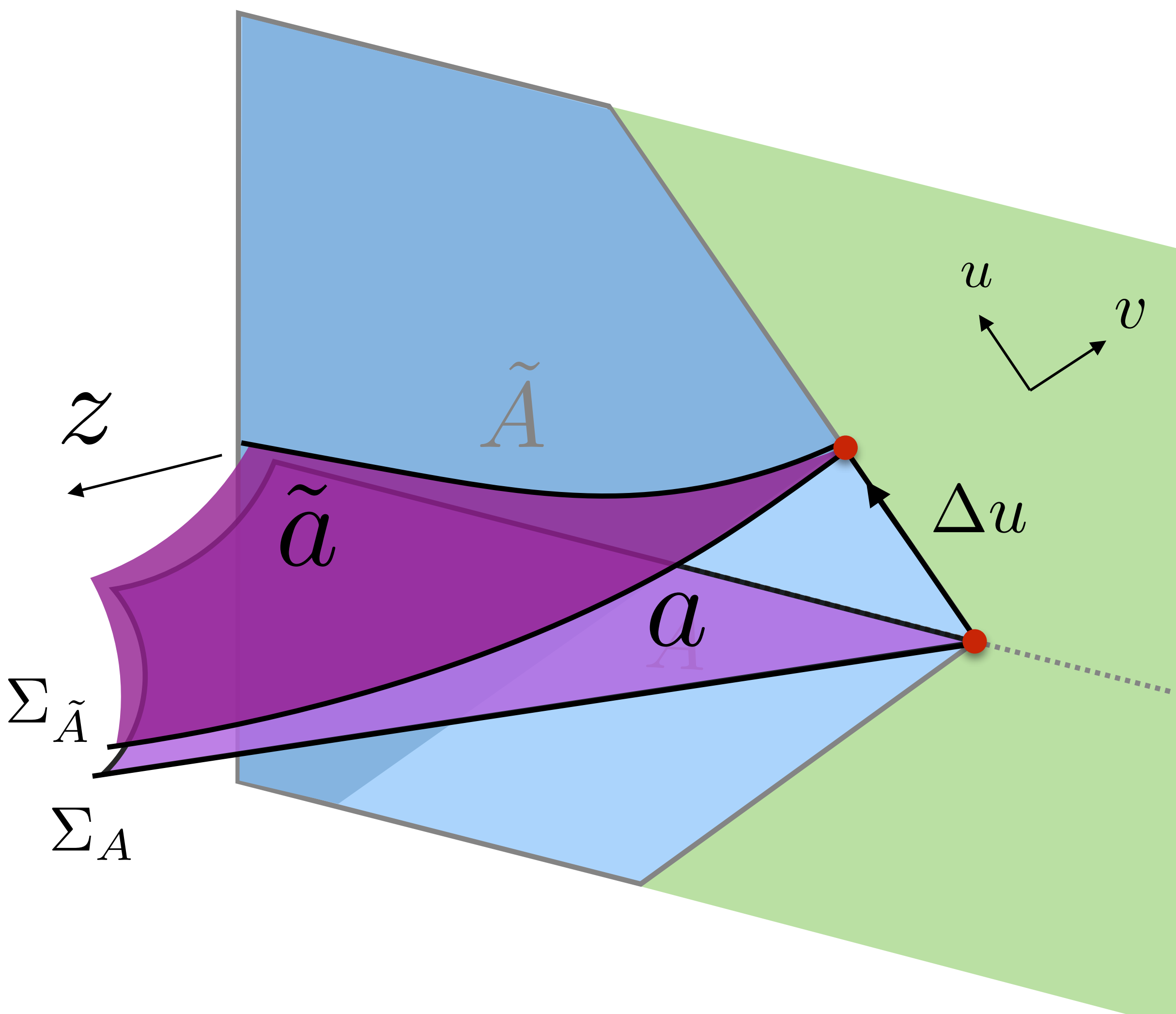

$\Sigma_{\tilde{A}}$ spacelike/null Σ_A

$z \rightarrow 0$

$\Delta u \rightarrow 0$

$$\langle T_{uu}\rangle_\psi - \partial_u^2 S_{EE} \geq 0$$

boundary QNEC

Plan of the talk:

- Review of AdS/CFT proofs (ANEC + QNEC)
- Summary of general field theory proofs (ANEC + QNEC)
- Bulk modular flow in AdS/CFT
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Proving ANEC in relativistic QFTs

$$\int_{-\infty}^{\infty} dx^+ \langle \hat{T}_{++} \rangle_\psi \geq 0$$

T. Faulkner, R. Leigh, O. Parrikar, H. Wang, 2016

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- difficult using conventional QFT techniques
- surprising origin in information theory
- manifested by probing the entanglement structure

Proving ANEC in relativistic QFTs

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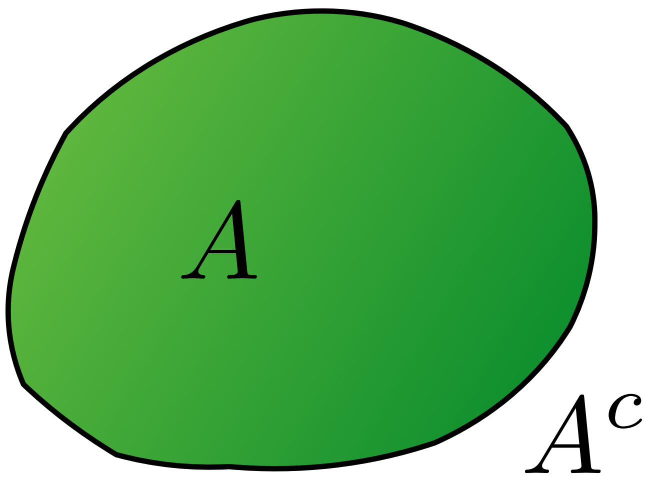
Modular Hamiltonian:

$$K_A^\Psi = -\ln \rho_A^\Psi \otimes \mathbb{1}_{A^c} + \mathbb{1}_A \otimes \ln \rho_{A^c}^\Psi = H_A^\Psi - H_{A^c}^\Psi$$

$$K_A^\Psi \;:\; \mathcal{H}_{\text{full}} \to \mathcal{H}_{\text{full}} \qquad\qquad K_A^\Psi |\Psi\rangle = 0$$



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Modular Hamiltonian:

$$K_A^\Psi = -\ln \rho_A^\Psi \otimes \mathbb{1}_{A^c} + \mathbb{1}_A \otimes \ln \rho_{A^c}^\Psi = H_A^\Psi - H_{A^c}^\Psi$$

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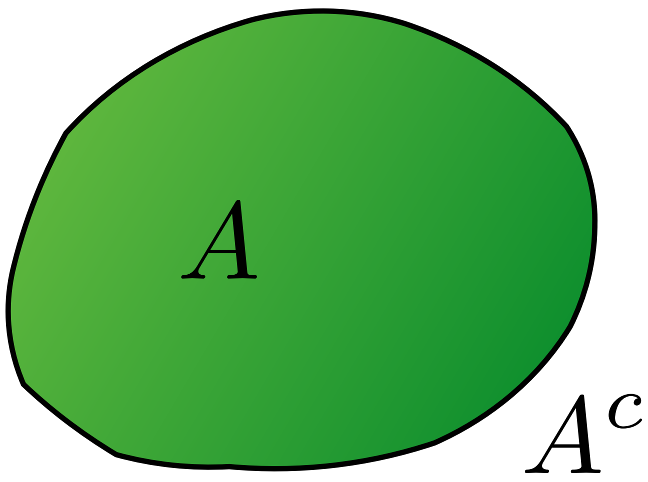

- encodes more detailed entanglement data
- in general, complicated and non-local
- simplifies in special cases

e.g. $\quad \Psi = |\text{vac}\rangle, \ A = \text{half-space}, \ K_A^\Psi = 2\pi \int d^{d-1}x \, x^1 T_{00} = \text{Rindler Hamiltonian}$

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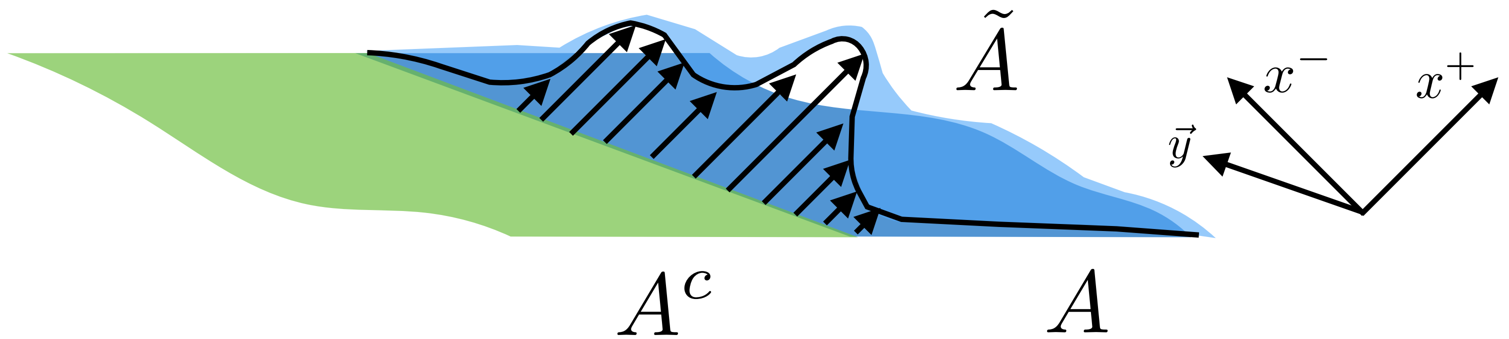
T. Faulkner, R. Leigh, O. Parrikar, H. Wang, 2016

Monotonicity property:

$$\tilde{A} = A + \vec{\xi}(y)$$



$$D(\tilde{A}) \subseteq D(A)$$

$$\Downarrow$$

$$\langle K^{\text{vac}}_{\tilde{A}} \rangle_\psi \leq \langle K^{\text{vac}}_{A} \rangle_\psi$$ where ψ is arbitrary

Proving ANEC in relativistic QFTs

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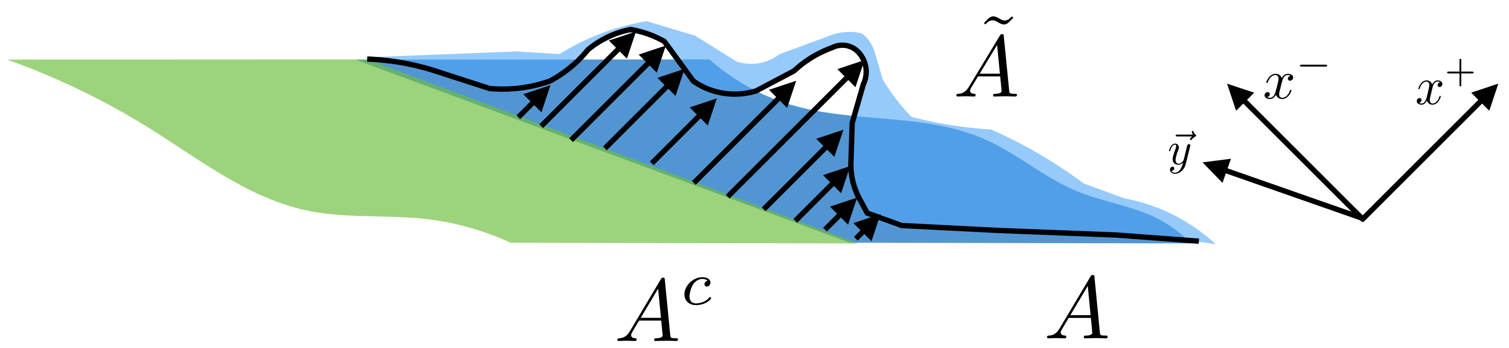
Monotonicity property: $\tilde{A} = A + \vec{\xi}(y)$



$$D(\tilde{A}) \subseteq D(A)$$

$\langle K_{\tilde{A}}^{\text{vac}} \rangle_\psi \leq \langle K_A^{\text{vac}} \rangle_\psi$ where ψ is arbitrary

Why? Monotonicity of relative entropy $S_A(\psi|\phi) = \mathrm{tr}\rho_A(\psi) \ln\left[\rho_A(\psi)/\rho_A(\phi)\right]$

measure of "distinguishability" $\longrightarrow$ $S_{\tilde{A}}(\psi|\phi) \leq S_A(\psi|\phi)$ for $D(\tilde{A}) \subseteq D(A)$

for special case of $|\phi\rangle = |\text{vac}\rangle$: $\langle K_{\tilde{A}}^{\text{vac}} \rangle_\psi \leq \langle K_A^{\text{vac}} \rangle_\psi$

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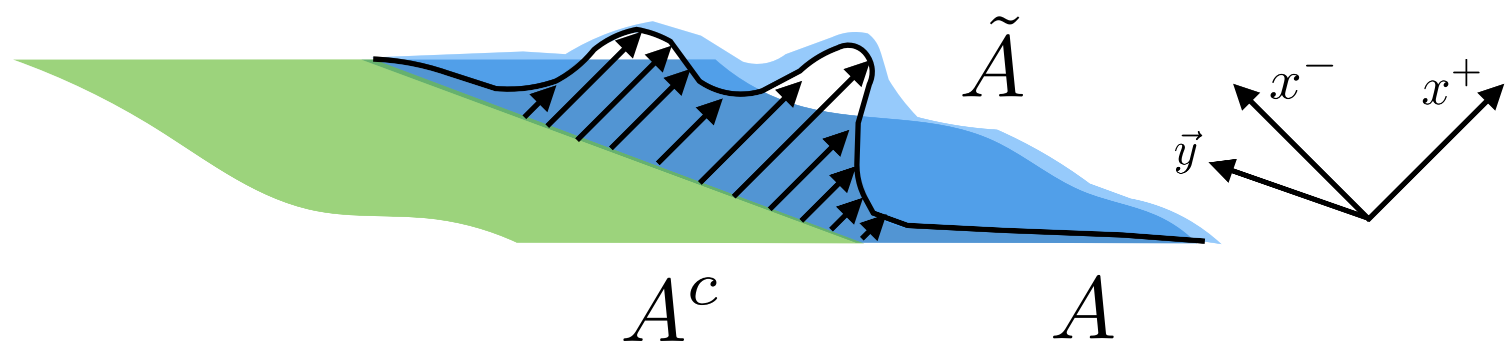
T. Faulkner, R. Leigh, O. Parrikar, H. Wang, 2016

Monotonicity property: $\tilde{A} = A + \vec{\xi}(y)$

$$D(\tilde{A}) \subseteq D(A)$$



$\langle K_{\tilde{A}}^{\text{vac}} \rangle_\psi \leq \langle K_A^{\text{vac}} \rangle_\psi$ where ψ is arbitrary

perturbation theory: $A =$ half-space, $K_A^{\text{vac}} =$ Rindler Hamiltonian

requiring $\langle K_{\tilde{A}}^{\text{vac}} \rangle_\psi \leq \langle K_A^{\text{vac}} \rangle_\psi$ for arbitrary null $\xi^+(\vec{y}) > 0$ "=" ANEC

Proving QNEC in general CFTs $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

Proving QNEC in general CFTs $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

- ANEC proof from entanglement structure
- alternative proof of ANEC from causality of correlation function

  T. Hartman, S. Kundu, A. Tajdini, 2016
- combine entanglement structure + causality?
- proof of QNEC (stronger conjecture)!

Proving QNEC in general CFTs $\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

causality of correlation function: $f(u,v) \propto \langle\psi|\mathcal{O}(u,v)\mathcal{O}(-u,-v)|\psi\rangle$

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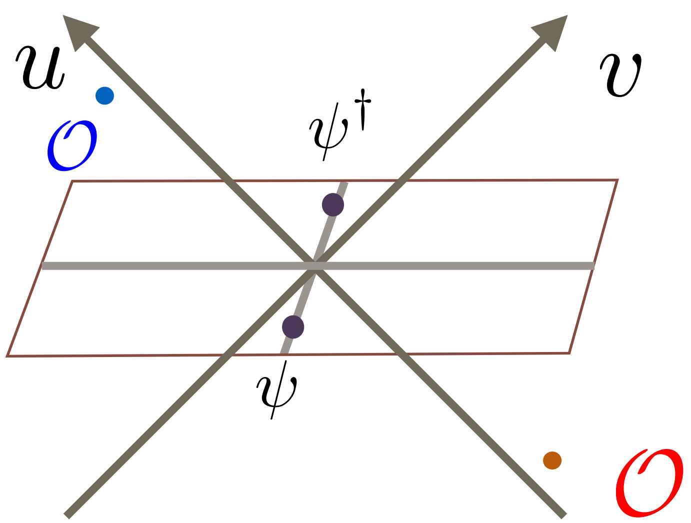

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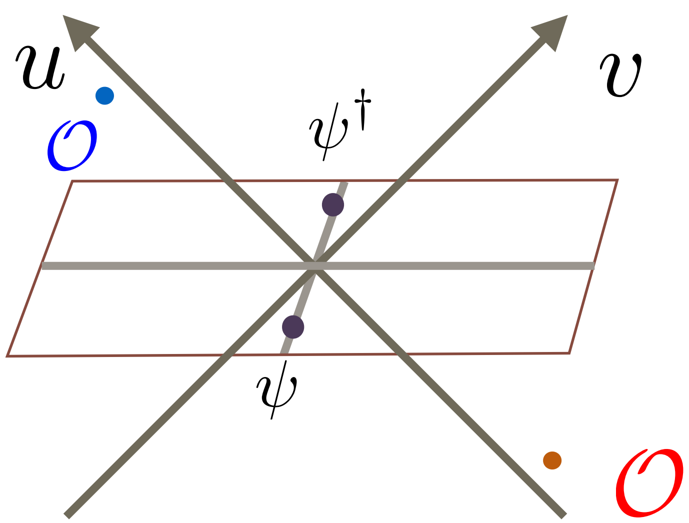

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"dress" the correlator to probe entanglement structure?

modular flow: $\mathcal{O} \to \mathcal{O}^A(s) \equiv e^{is\, K_A^\psi} \mathcal{O} e^{-is\, K_A^\psi}$

in general: highly non-local!

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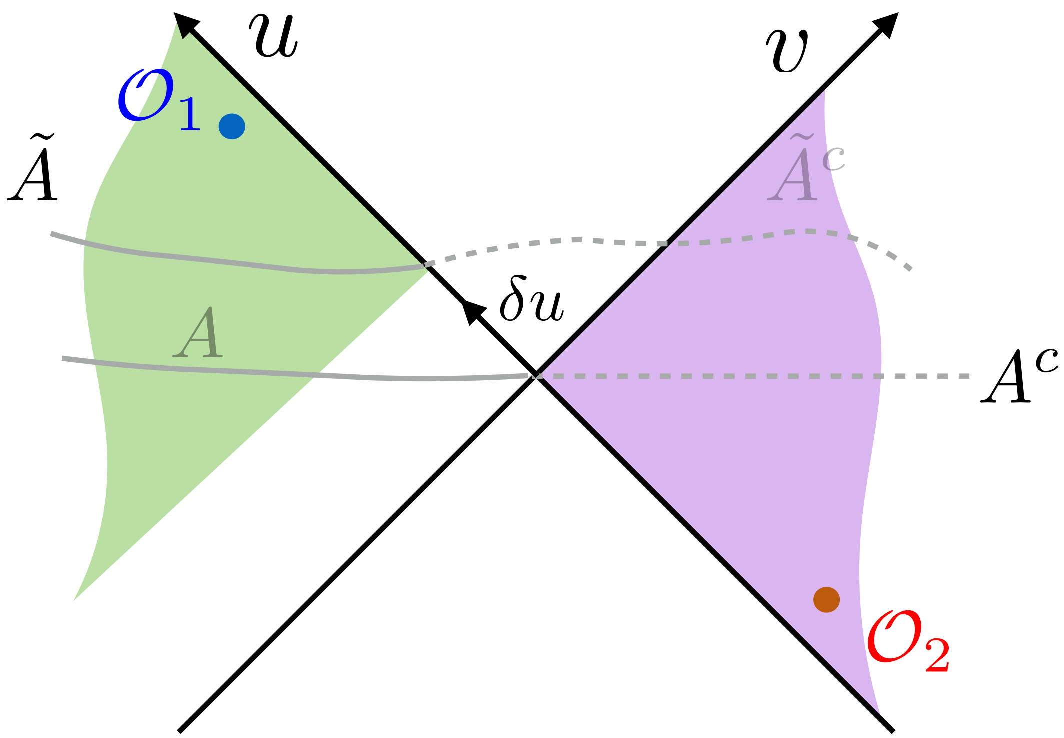

consider:

$$f(s) = \mathcal{N}^{-1}\langle\psi|\mathcal{O}_1^{\tilde{A}}(s)\mathcal{O}_2^{A}(s)|\psi\rangle$$

$$\mathcal{O}_1^{\tilde{A}}(s) = e^{is\,K^{\psi}_{\tilde{A}}}\mathcal{O}_1 e^{-is\,K^{\psi}_{\tilde{A}}}$$

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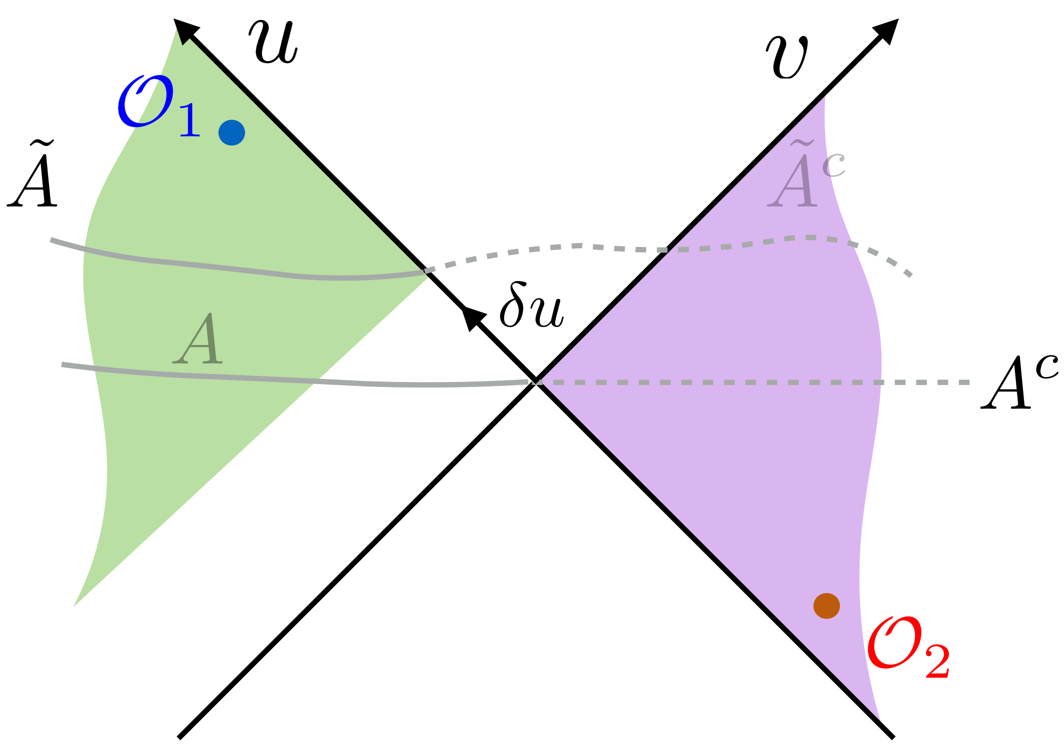

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Tomita-Takesaki theory (in algebraic QFT):

$$\mathcal{O} \in \mathcal{M}_A \to \mathcal{O}^A(s) \in \mathcal{M}_A, \quad s \in \mathbb{R}$$

$\mathcal{M}_A$: von Neumann algebra associated with A, i.e. operators supported in D(A)

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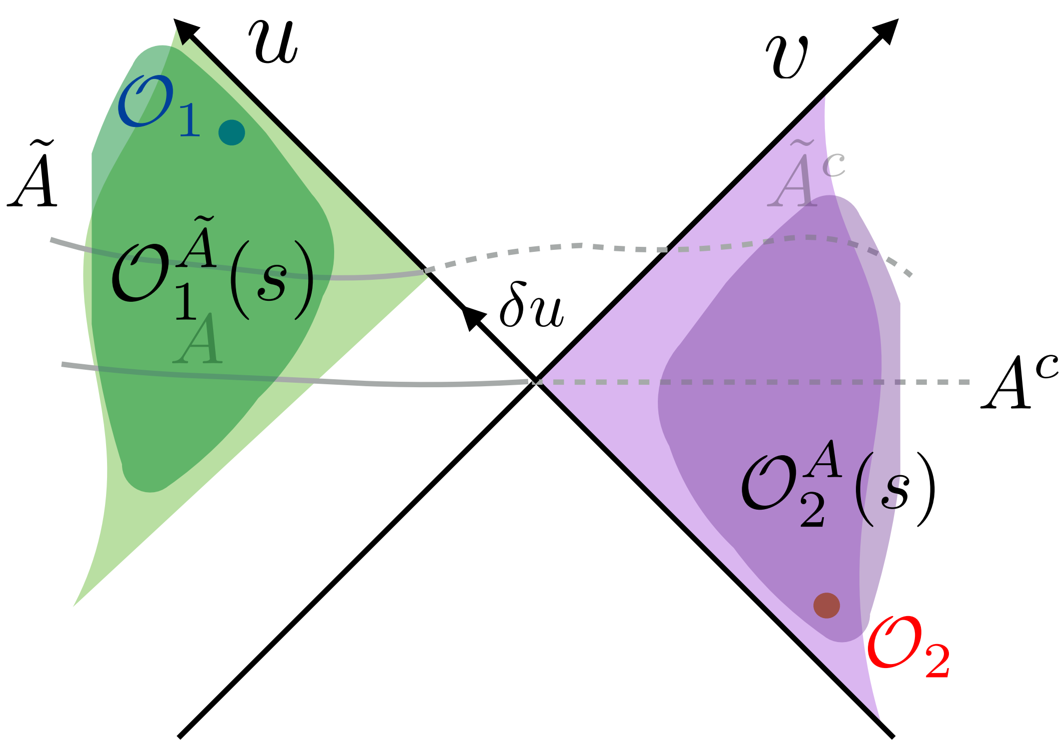
S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017



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Tomita-Takesaki theory (in algebraic QFT):

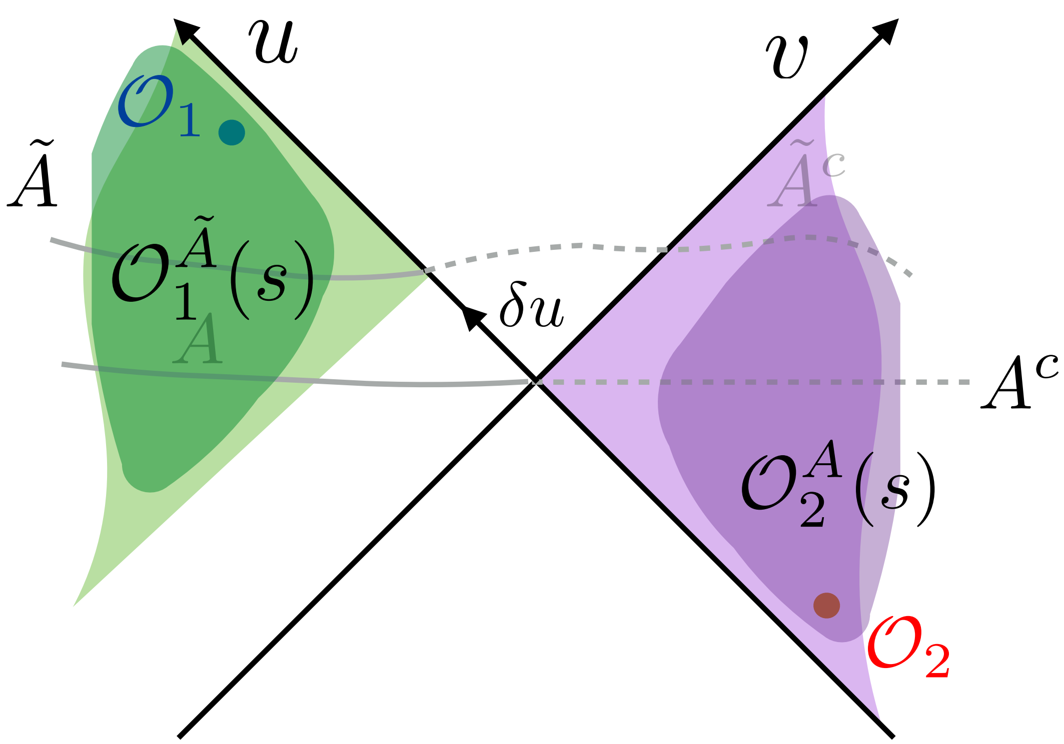
$\mathcal{O}_1^{\tilde{A}}(s)$ is supported only in $D(\tilde{A})$

$\mathcal{O}_2^{A}(s)$ is supported only in $D(A^c)$

Proving QNEC in general CFTs

$$\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017



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$$\left[\,\mathcal{O}_1^{\tilde{A}}(s)\,,\,\mathcal{O}_2^{A}(s)\,\right] = 0 \quad \text{for} \quad s \in \mathbb{R}$$

a subtler notion of causality:

hidden in entanglement structure!

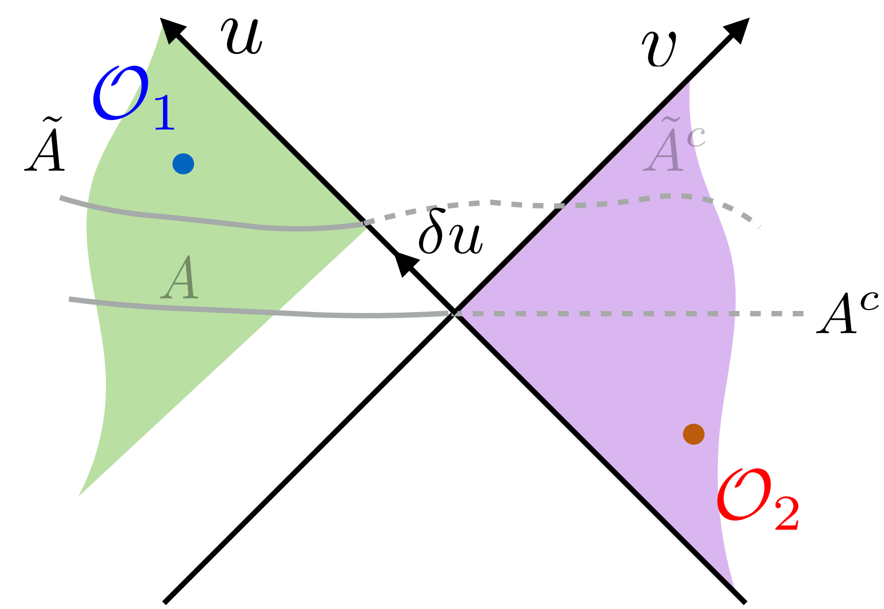
Proving QNEC in general CFTs $\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

Outline of the proof:

1. Unitarity + Cauchy-Schwarz inequality:

$$\mathrm{Re} f(s) \leq 1, \ \mathrm{Im}\ s = \pm\pi/2$$



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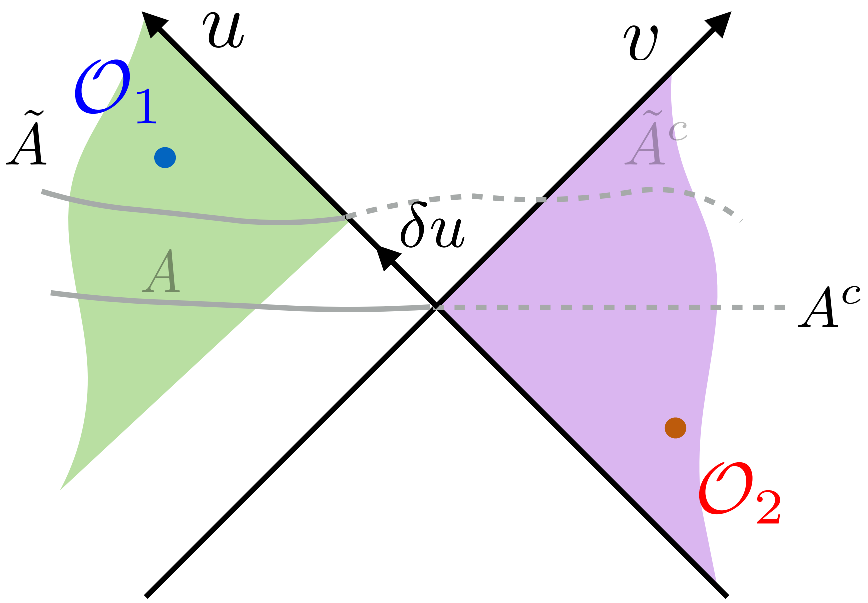

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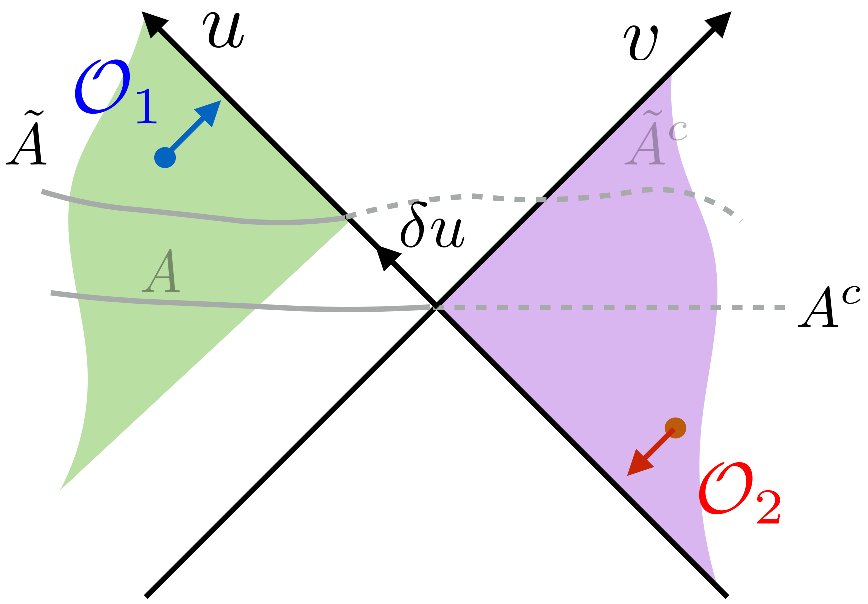

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3. Light-cone limit expansion: $v \to 0, \ u$ fixed

$$f(s) = 1 + C_T^{-1} e^s u(-uv)^{\frac{d-2}{2}} \mathcal{I}_Q + \ldots$$

$$\mathcal{I}_Q = \int_0^{\delta u} du' T_{uu}(u') + \left(\frac{\delta S_{EE}(A)}{\delta u} - \frac{\delta S_{EE}(\tilde{A})}{\delta u} \right)$$

Proving QNEC in general CFTs $\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$

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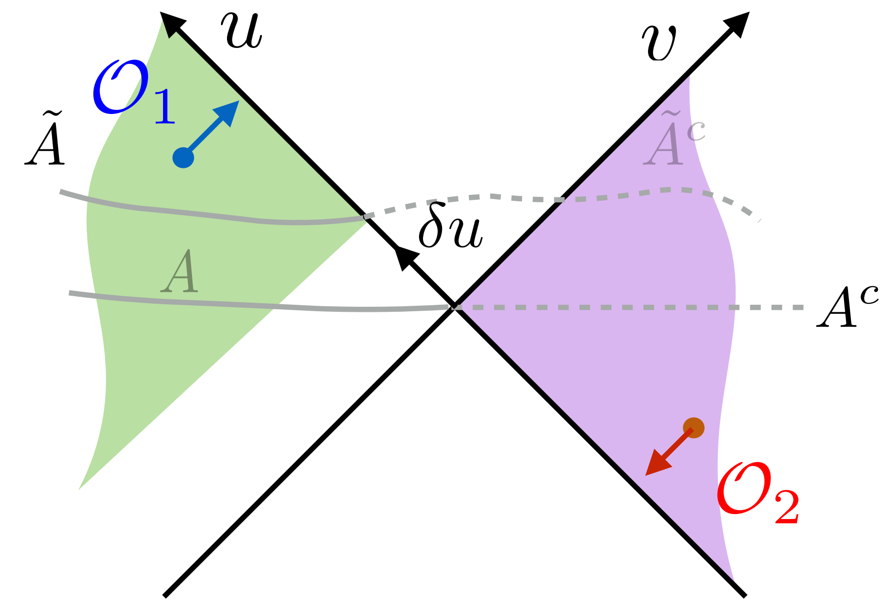

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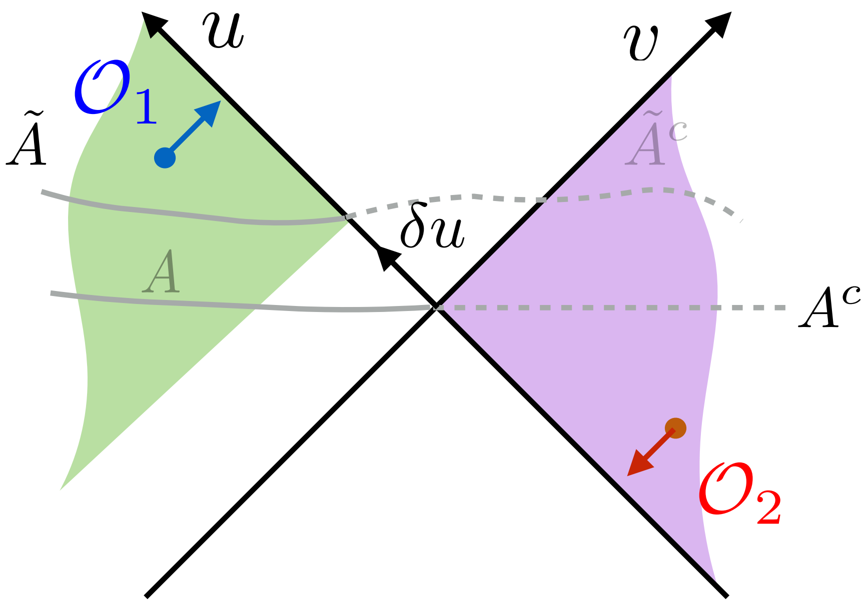

$$\mathcal{I}_Q = \int_0^{\delta u} du' T_{uu}(u') + \left(\frac{\delta S_{EE}(A)}{\delta u} - \frac{\delta S_{EE}(\tilde{A})}{\delta u}\right) \approx \delta u\left(\langle T_{uu}\rangle_\psi - \partial_u^2 S_{EE}\right) \geq 0$$

$$(\lim \delta u \to 0) \to \left(\langle T_{uu}\rangle_\psi - \partial_u^2 S_{EE}\right) \geq 0 \qquad \text{QNEC} ✓$$

Plan of the talk:

- Review of AdS/CFT proofs (ANEC + QNEC)
- Summary of general field theory proofs (ANEC + QNEC)
- Bulk modular flow in AdS/CFT
- Conclusion/outlooks

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

- in holography, EWN near boundary = boundary QNEC

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- understand this connection more explicitly
- a concrete step: bulk approach for computing $f(s)$

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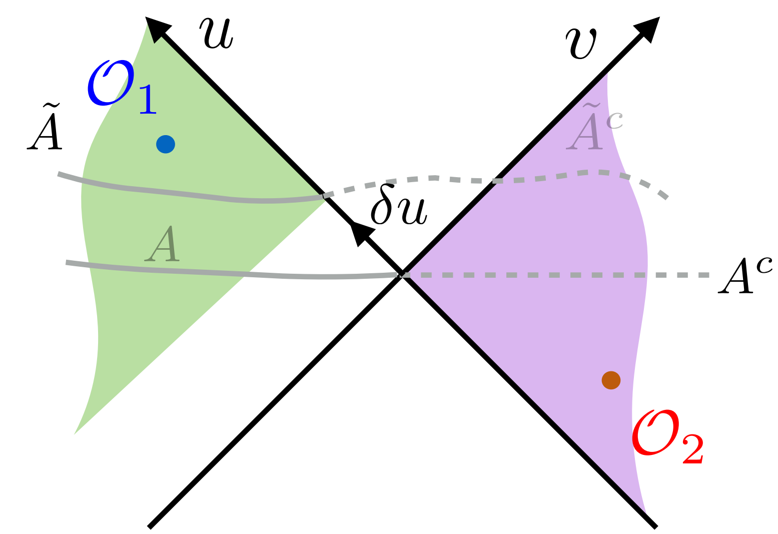
Revisit $f(s) \propto \langle\psi|\mathcal{O}_1^{\tilde{A}}(s)\mathcal{O}_2^{A}(s)|\psi\rangle$



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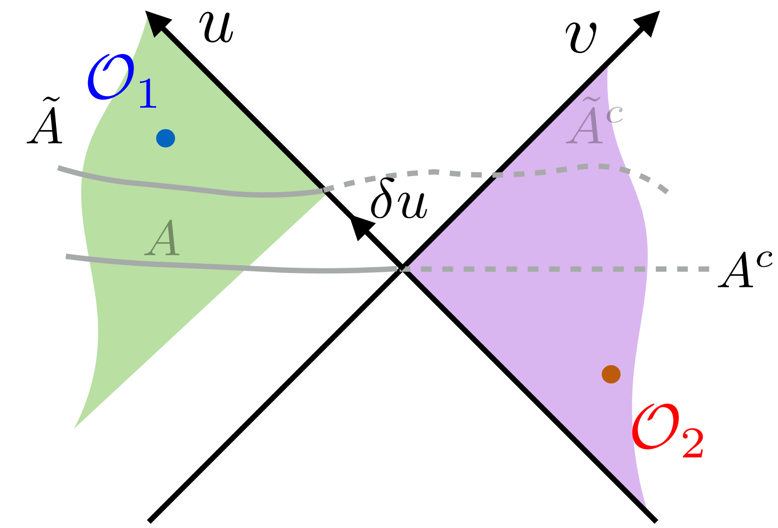

from "Heisenberg" to "Schrodinger" picture:

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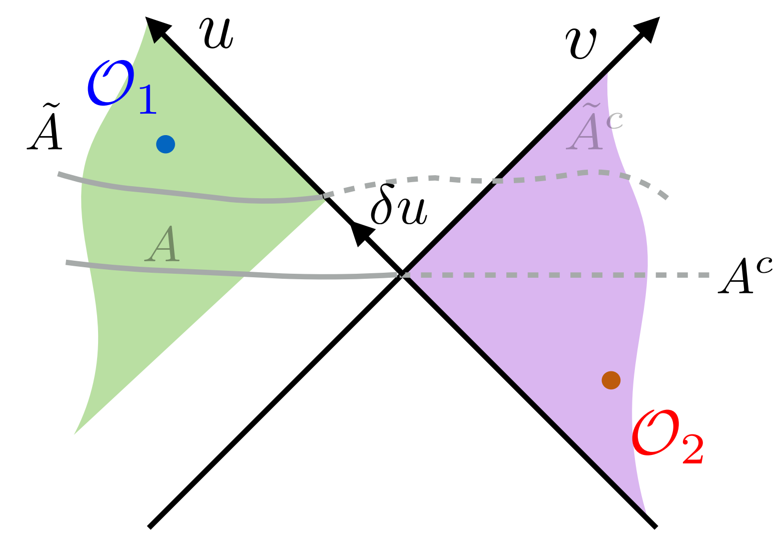

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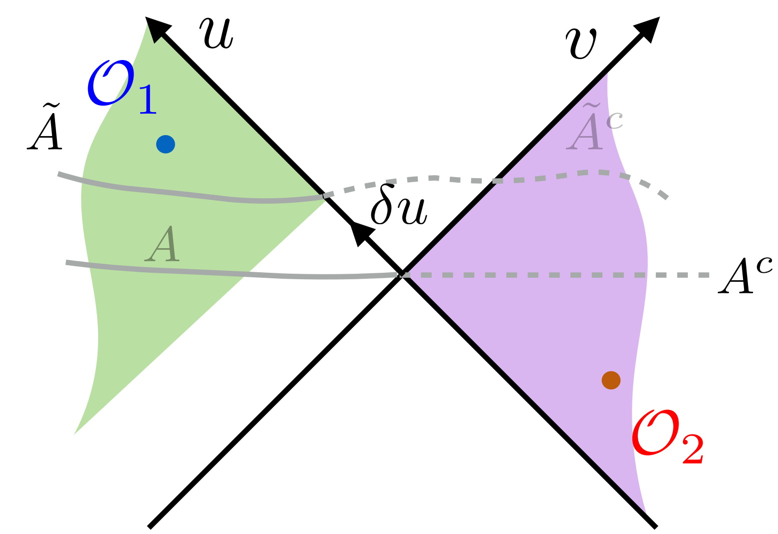

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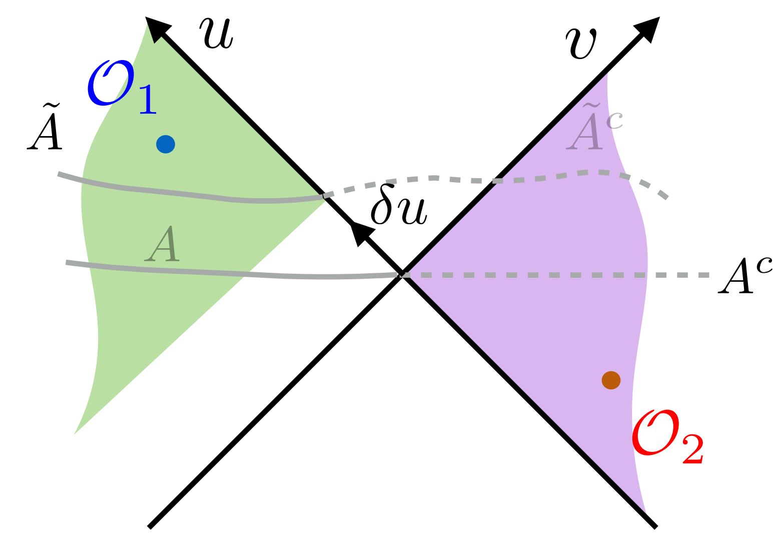

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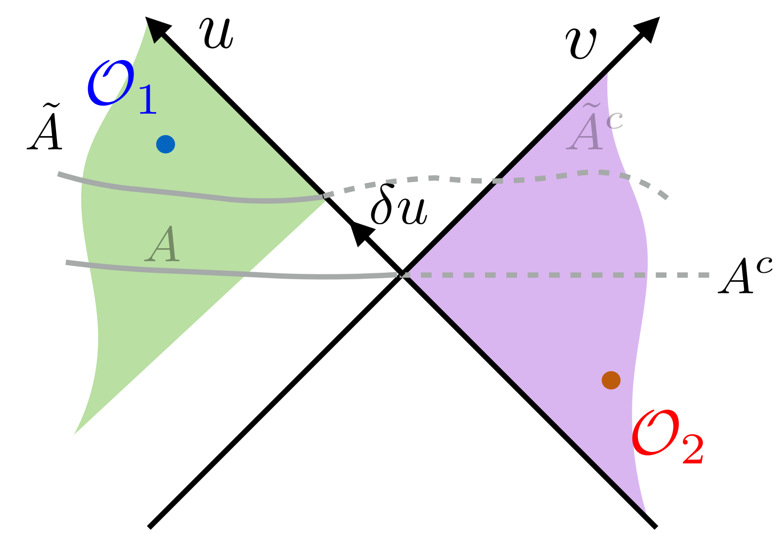

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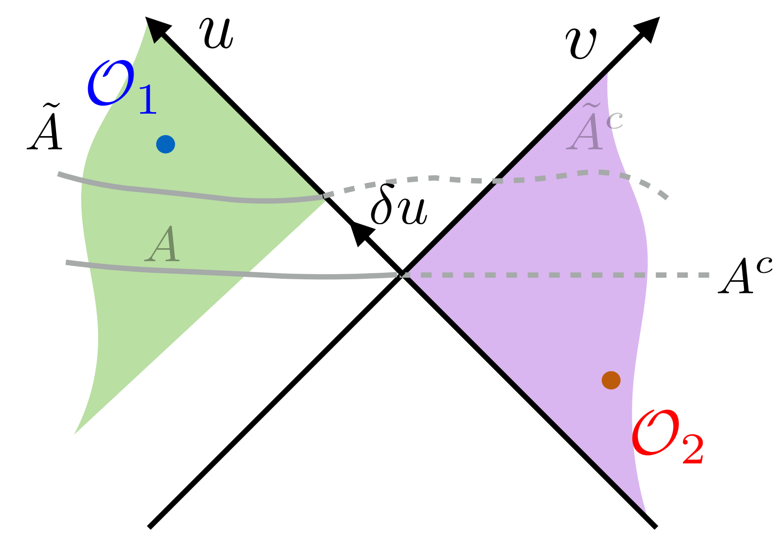

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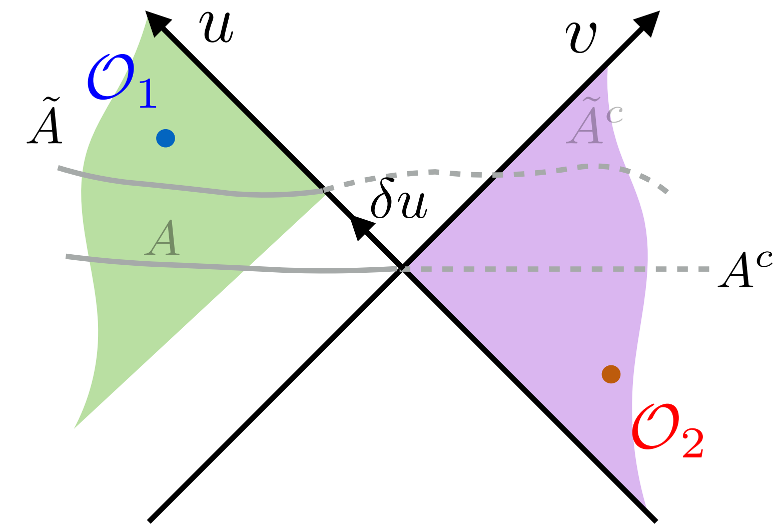

to use AdS/CFT, consider:

- in a holographic CFT
- bulk dual of $|\psi\rangle$ has smooth geometry
- conformal dimension Δ of $\mathcal{O}_{1,2} : 1 \ll \Delta \ll \ell_{AdS}/\ell_{plank}$

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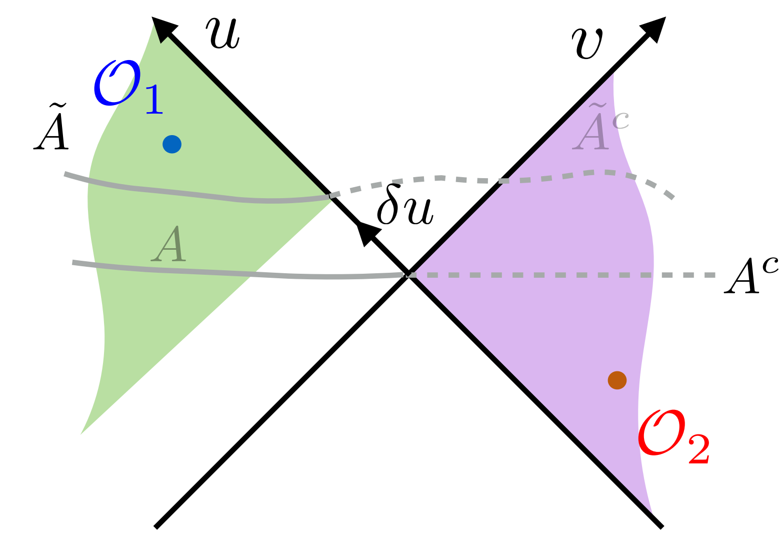
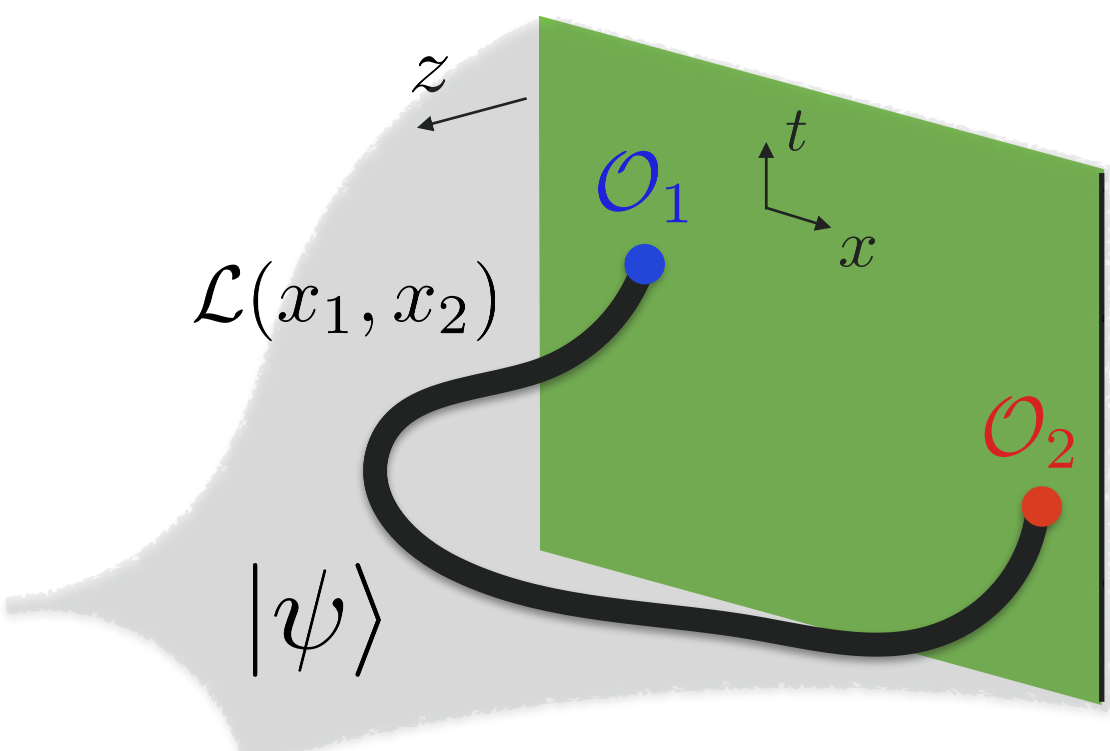


Geodesic approximation:

$$\langle \psi | \mathcal{O}_1 \mathcal{O}_2 | \psi \rangle$$

$$\approx \exp\left[-m\mathcal{L}(x_1, x_2)\right]$$



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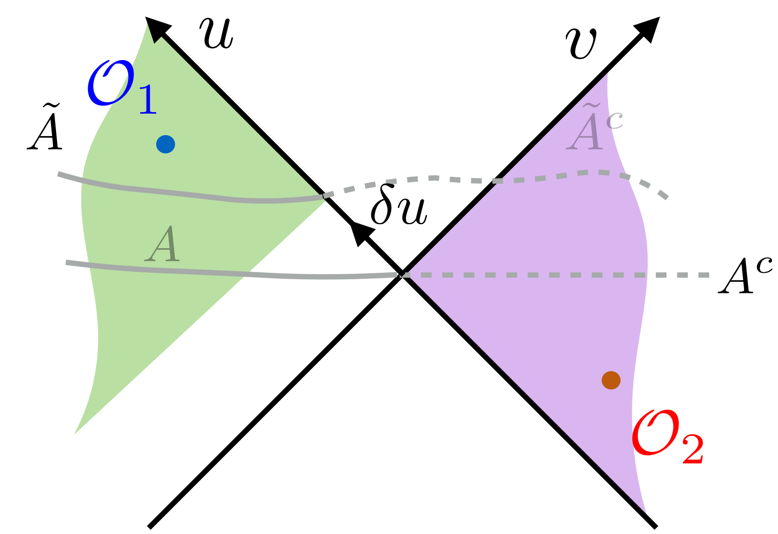
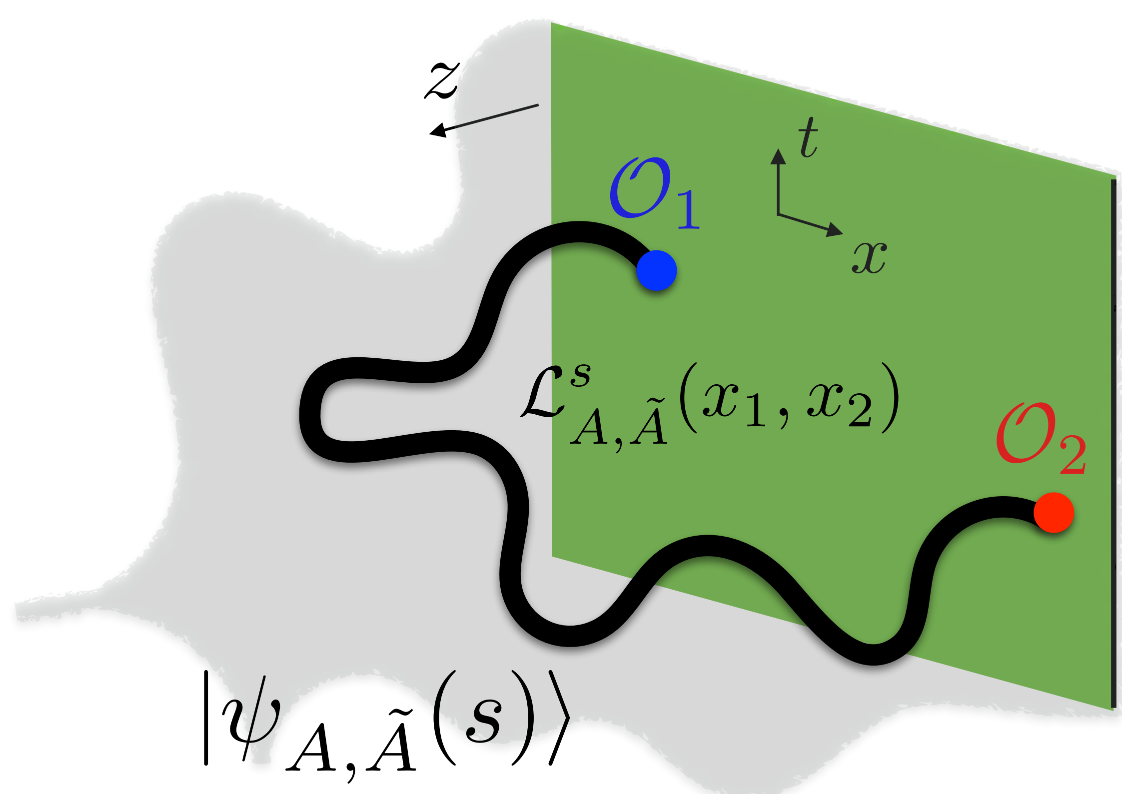


Geodesic approximation:

$$\langle \psi_{A,\tilde{A}}(s) | \, \mathcal{O}_1 \, \mathcal{O}_2 \, | \psi_{A,\tilde{A}}(s) \rangle$$

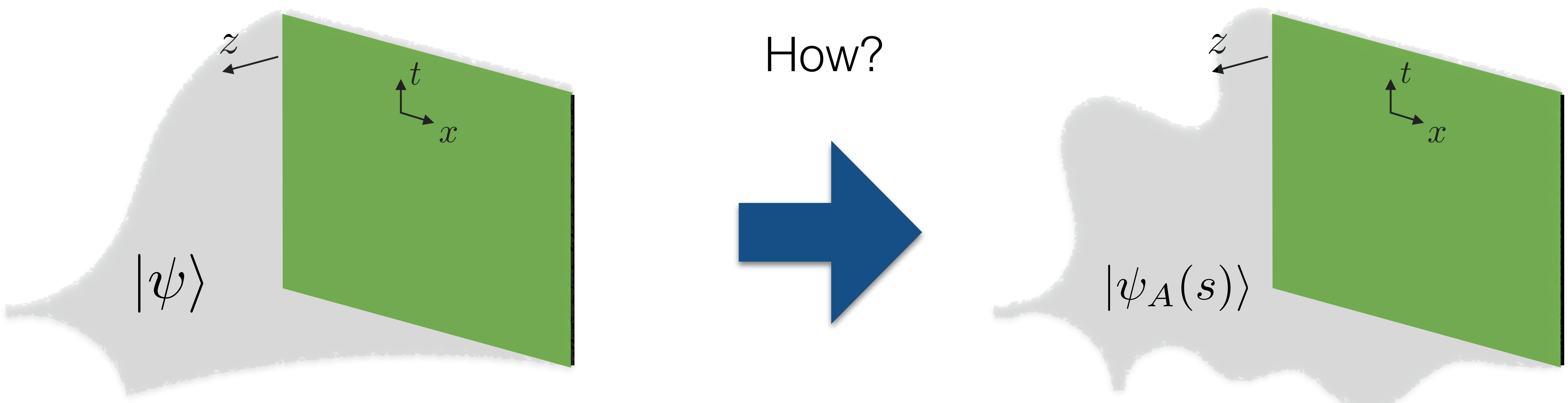
$$\approx \exp\left[-m \mathcal{L}^s_{A,\tilde{A}}(x_1, x_2) \right]$$



Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

consider a simpler case: $|\psi_A(s)\rangle = e^{isH_A^\psi}|\psi\rangle$ i.e. "single modular flow"



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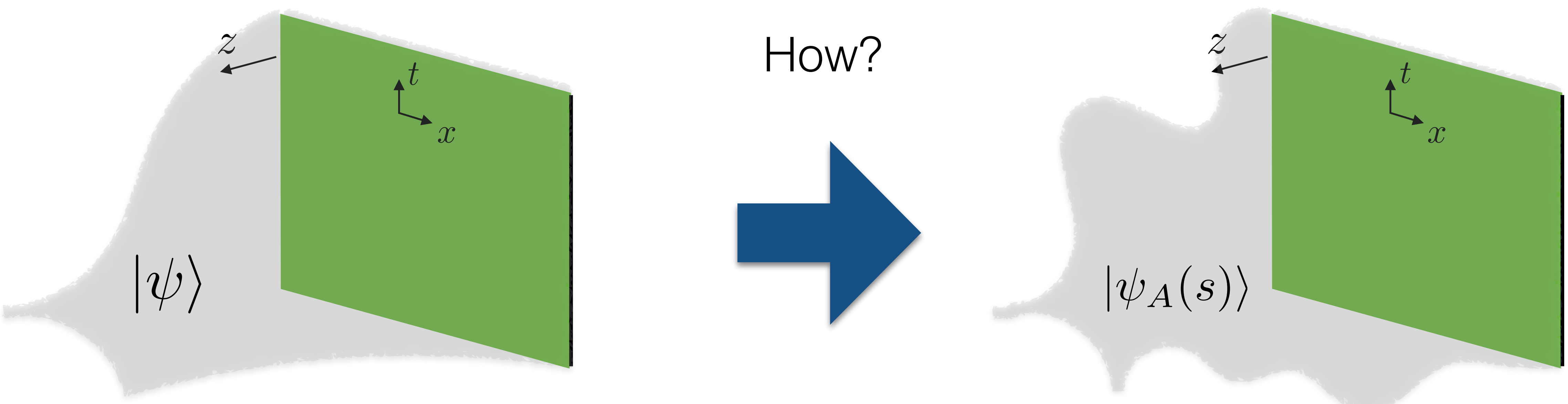

hint: for any $\mathcal{O}_A$ supported only in $D(A)$: $\langle \mathcal{O}_A \rangle_{\psi_A(s)} = \langle \mathcal{O}_A \rangle_\psi$

$$\langle \psi_A(s)|\mathcal{O}_A|\psi_A(s)\rangle = \langle \psi|e^{-isH_A^\psi}\mathcal{O}_A e^{isH_A^\psi}|\psi\rangle = \langle \psi|e^{-isH_{A^c}^\psi}\mathcal{O}_A e^{isH_{A^c}^\psi}|\psi\rangle$$

$$= \langle \psi|e^{-isH_{A^c}^\psi} e^{isH_{A^c}^\psi}\mathcal{O}_A|\psi\rangle = \langle \psi|\mathcal{O}_A|\psi\rangle$$

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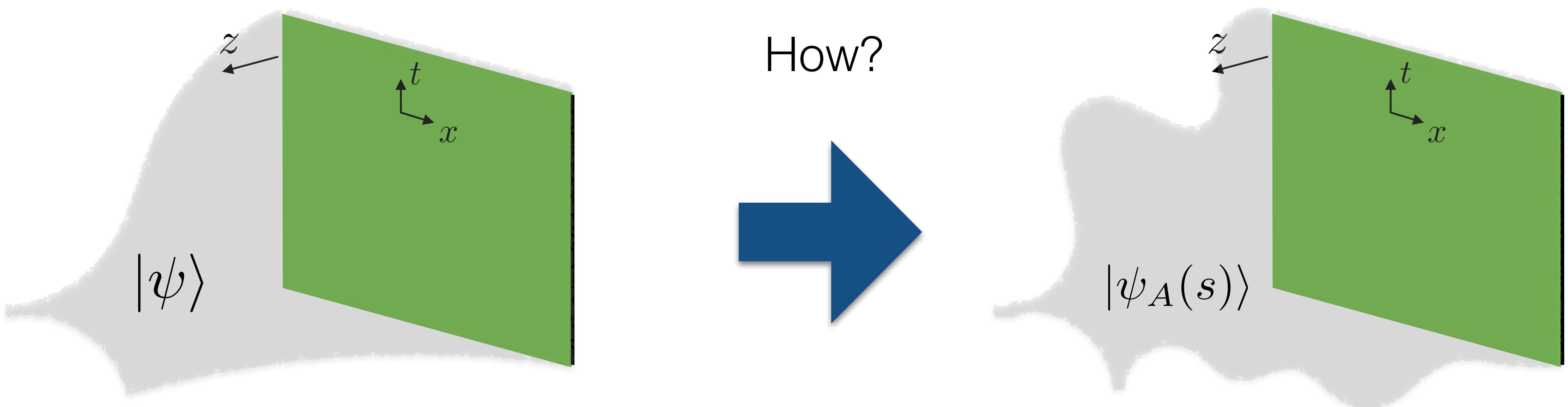


hint: for any $\mathcal{O}_A$ supported only in $D(A)$: $\langle \mathcal{O}_A \rangle_{\psi_A(s)} = \langle \mathcal{O}_A \rangle_\psi$

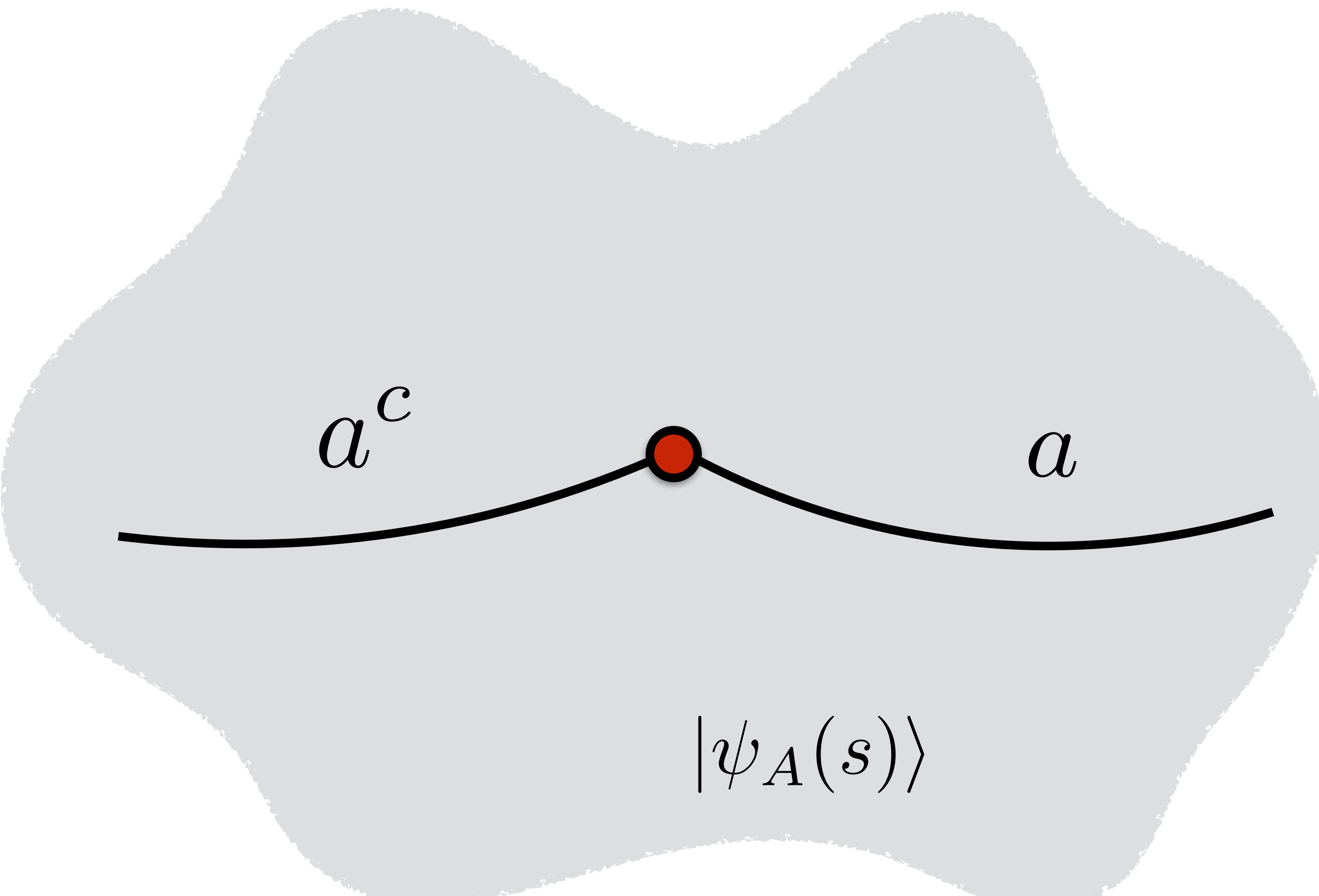
similarly,

for any $\mathcal{O}_{A^c}$ supported only in $D(A^c)$: $\langle \mathcal{O}_{A^c} \rangle_{\psi_A(s)} = \langle \mathcal{O}_{A^c} \rangle_\psi$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

entanglement wedge reconstruction: $D(a)\text{``}\approx\text{''}D(A),\ D(a^c)\text{``}\approx\text{''}D(A^c)$



Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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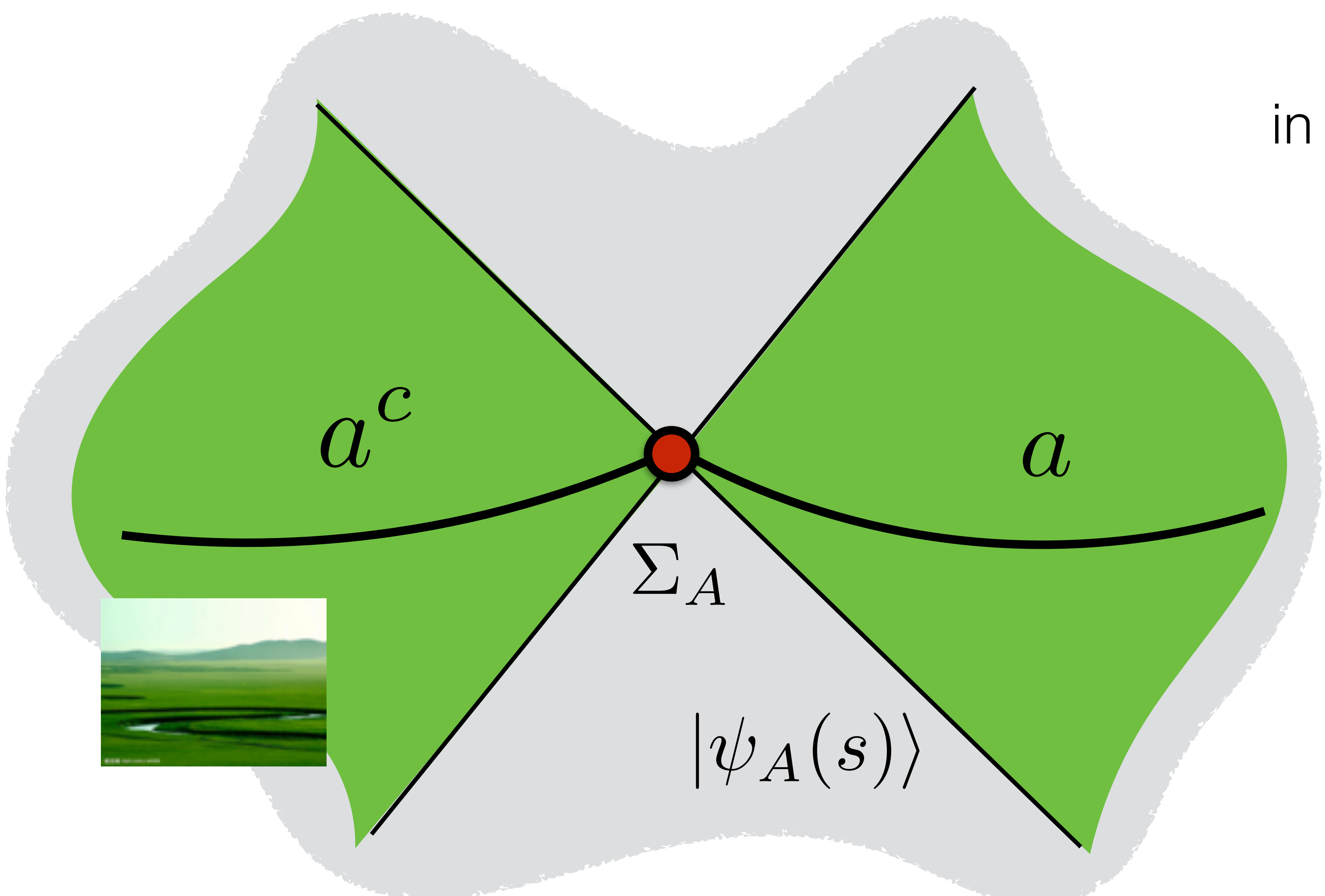


in entanglement wedges :

$$"\psi_A(s) \equiv \psi"$$

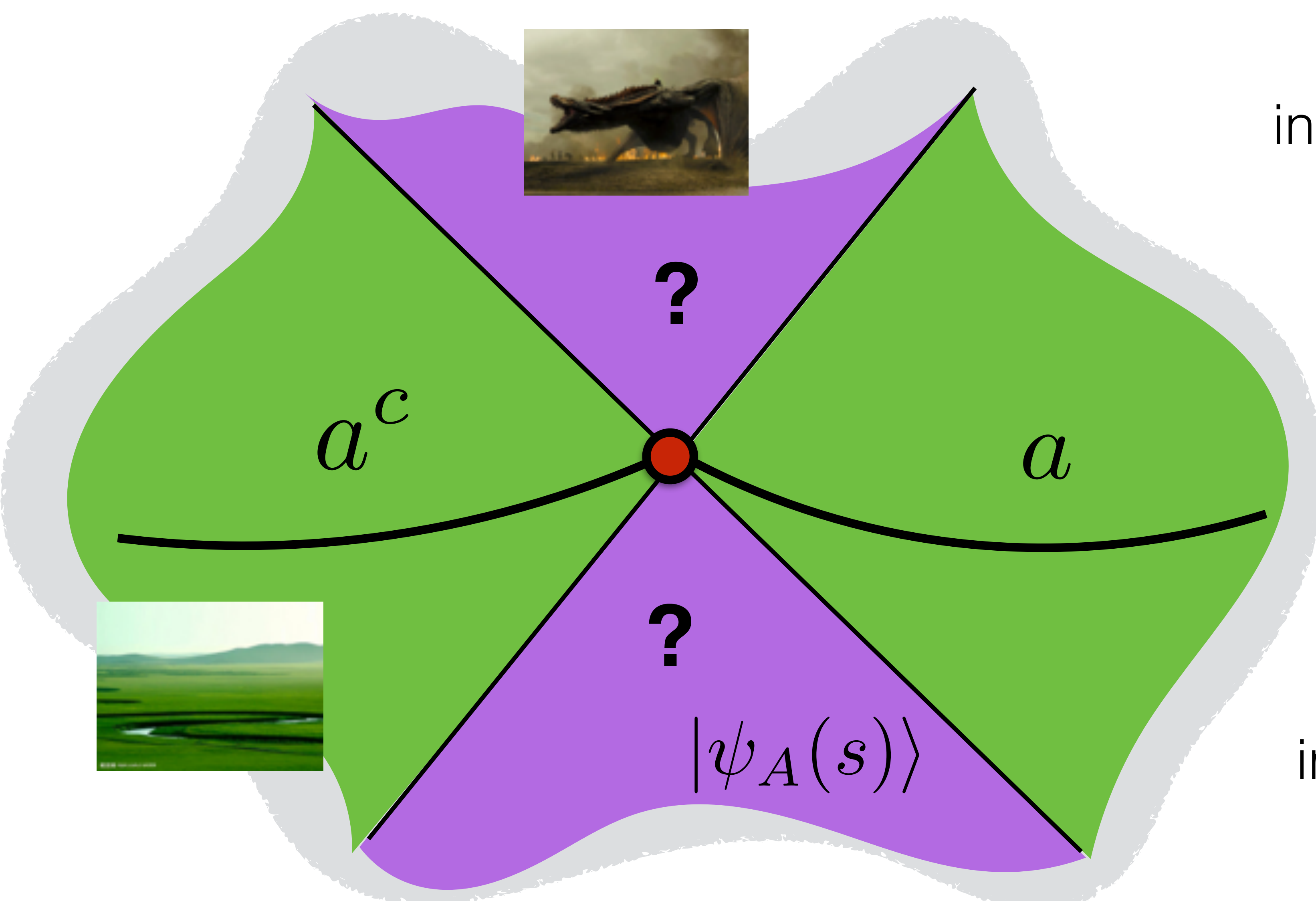
e.g.

- same metric
- same bulk fields, etc

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

entanglement wedge reconstruction: $D(a)\text{“}\approx\text{”}D(A),\ D(a^c)\text{“}\approx\text{”}D(A^c)$



in **entanglement wedges** :

$$''\psi_A(s) \equiv \psi''$$

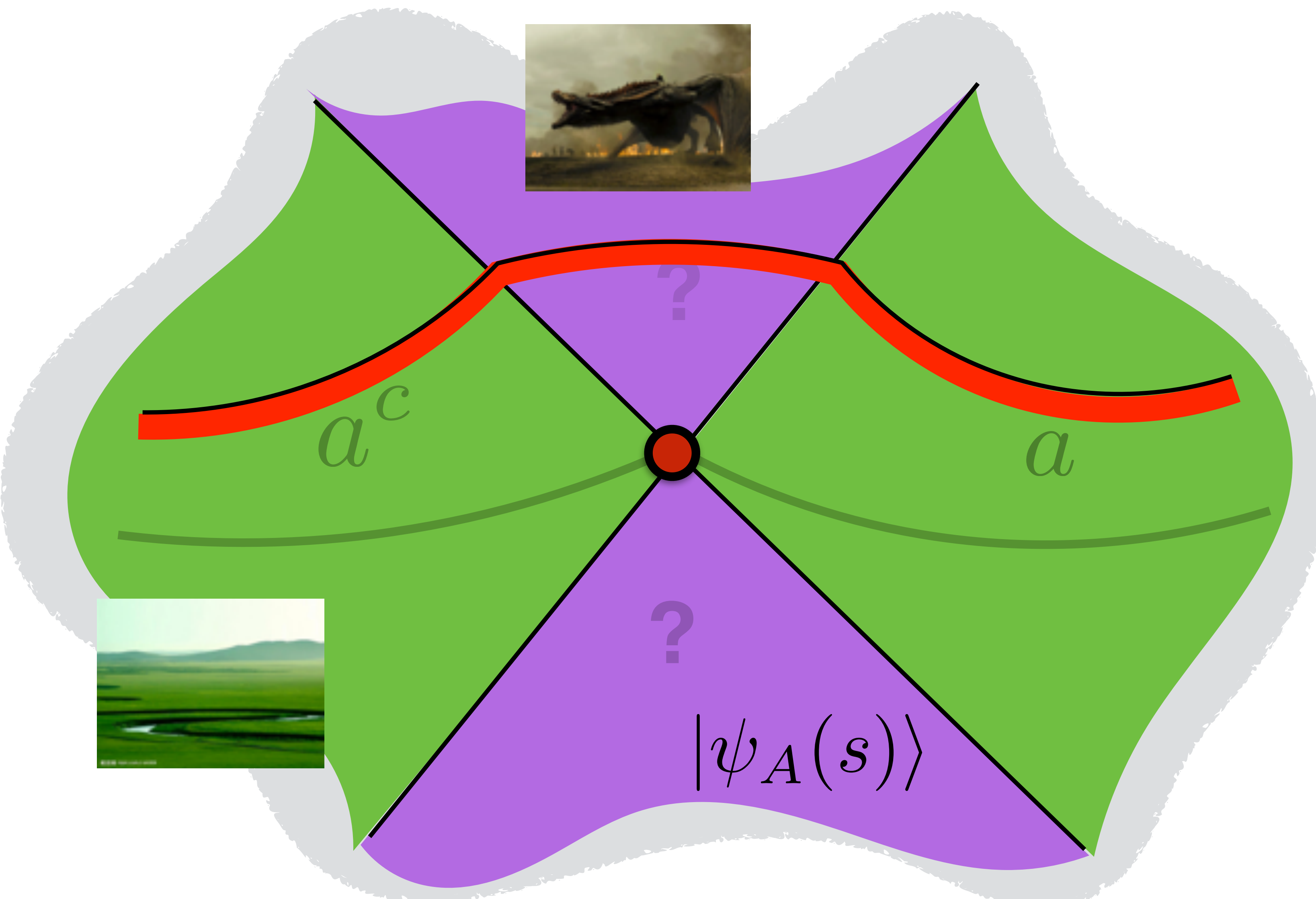
in **“Milne” wedges** :

unknown, possibly with kinks/singularities

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

entanglement wedge reconstruction: $D(a)``\approx" D(A),\ D(a^c)``\approx" D(A^c)$



geodesic: a function of $\{x_1, x_2, s\}$

generic geodesics pass through both the entanglement and "Milne" wedges

we don't know what to do…

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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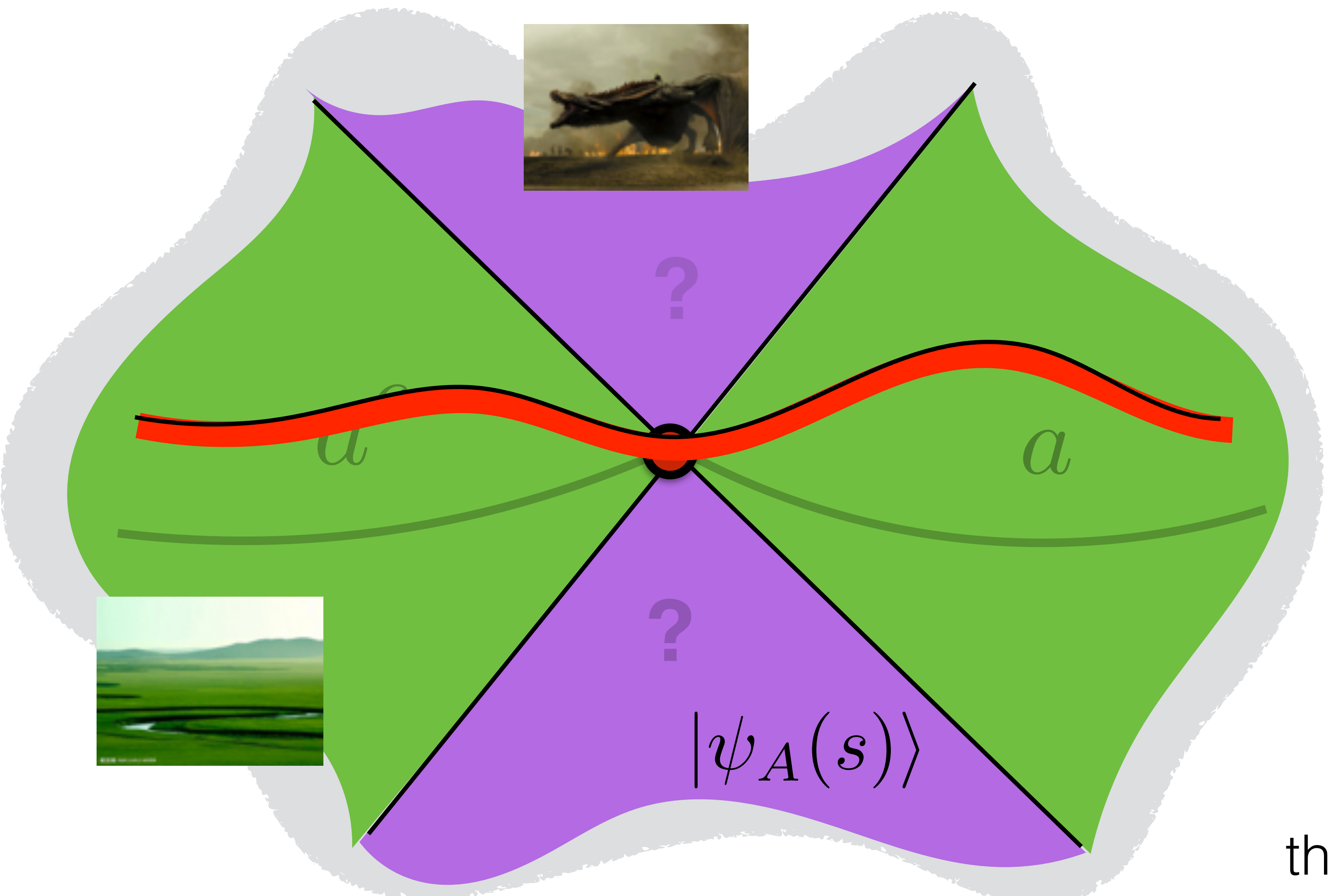


geodesic: a function of $\{x_1, x_2, s\}$

if we fine-tune one of the parameters:

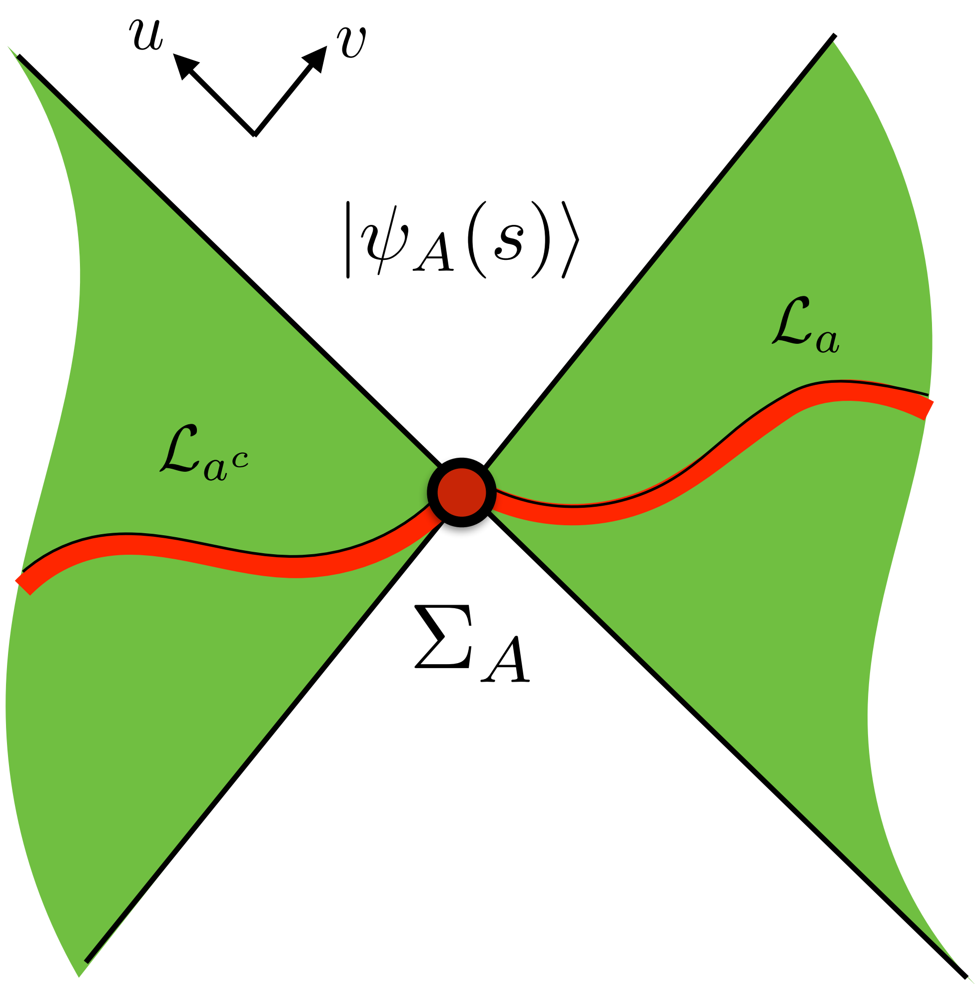
e.g. $s = s(x_1, x_2)$

the geodesic avoids the Milne wedge, passes through Σ_A

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

So, what do we know about geodesics in the entanglement wedges (EW)?



- each segment $\{\mathcal{L}_a, \mathcal{L}_{a^c}\}$ is a geodesic in the original geometry $|\psi\rangle$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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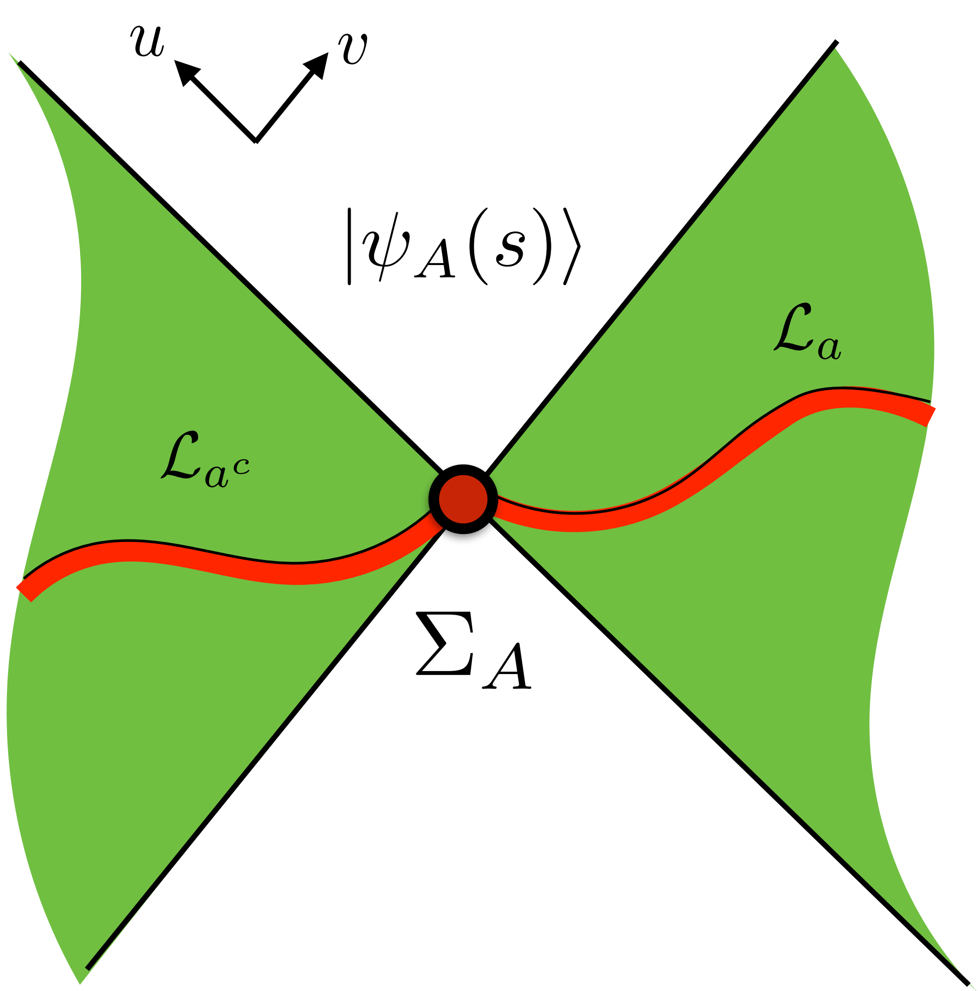

- each segment $\{\mathcal{L}_a, \mathcal{L}_{a^c}\}$ is a geodesic in the original geometry $|\psi\rangle$.
- modular flow affects the matching condition at Σ_A .

Bulk modular flow in AdS/CFT

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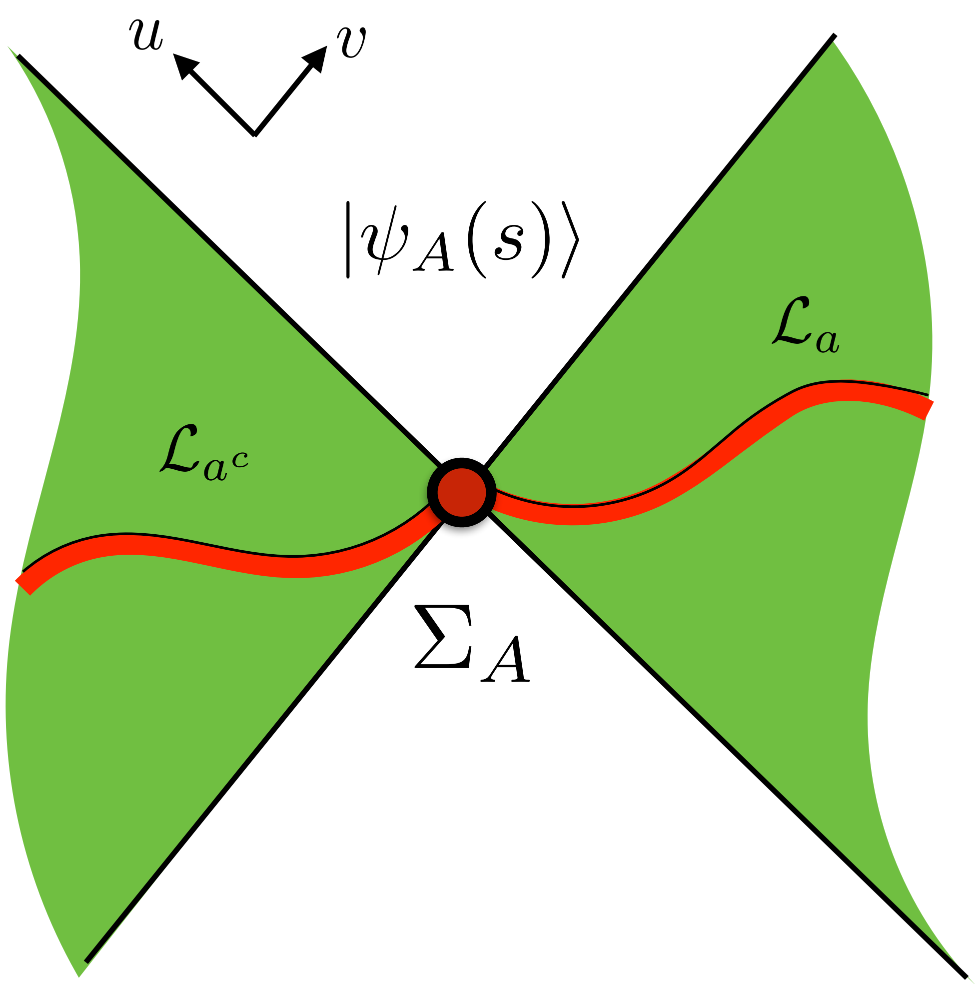

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Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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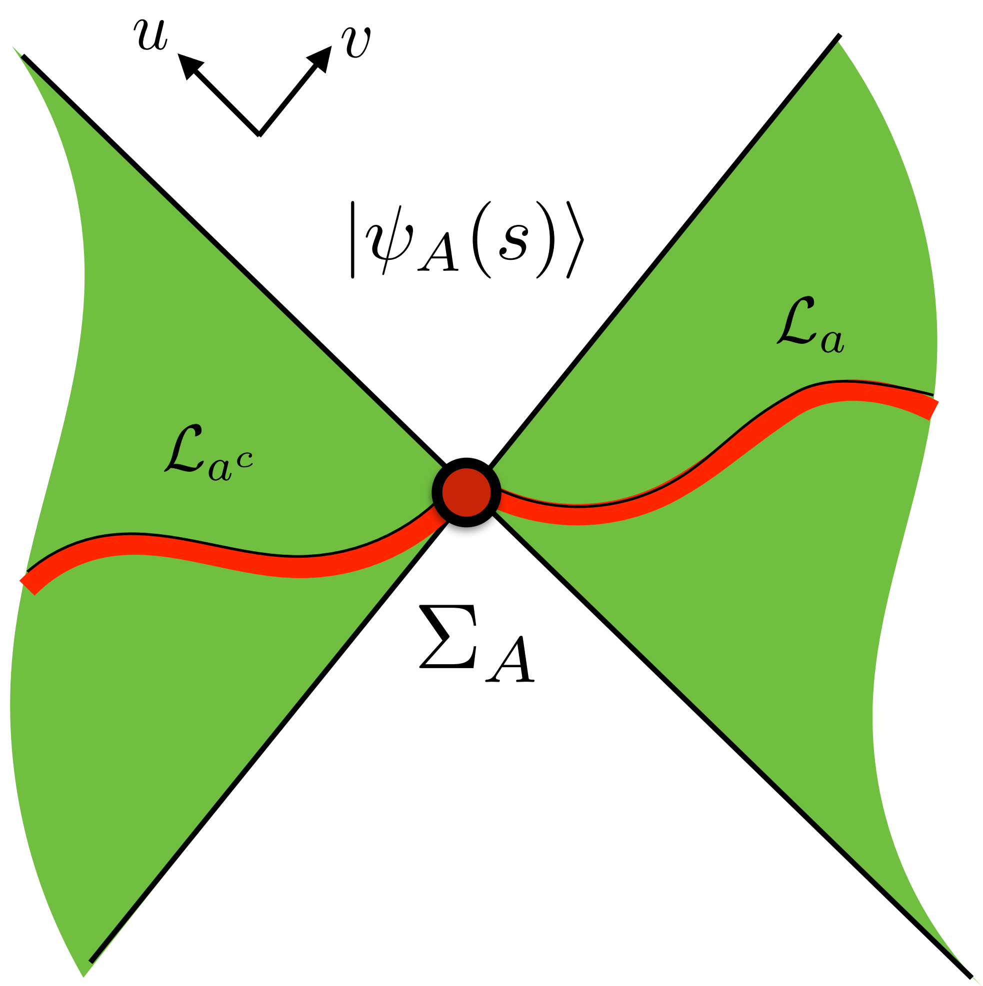

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Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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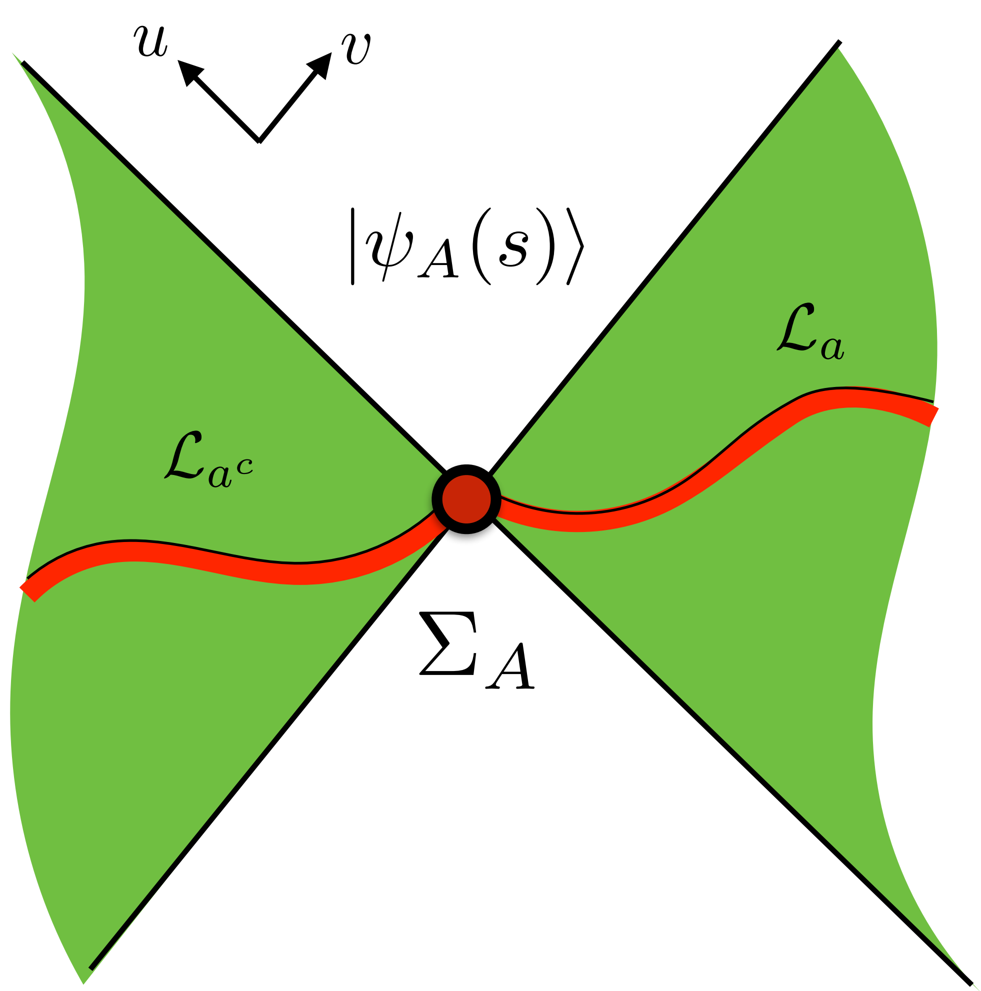

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- $\hat{A}$ is a constant in EW, $e^{isH_A^\psi(bdry)} \propto e^{isH_a^\psi(bulk)}$.
- bulk theory free (leading ordering 1/N): close to Σ_A , $H_a^\psi(bulk)$ acts like bulk Rindler Hamiltonian and generates boosts.

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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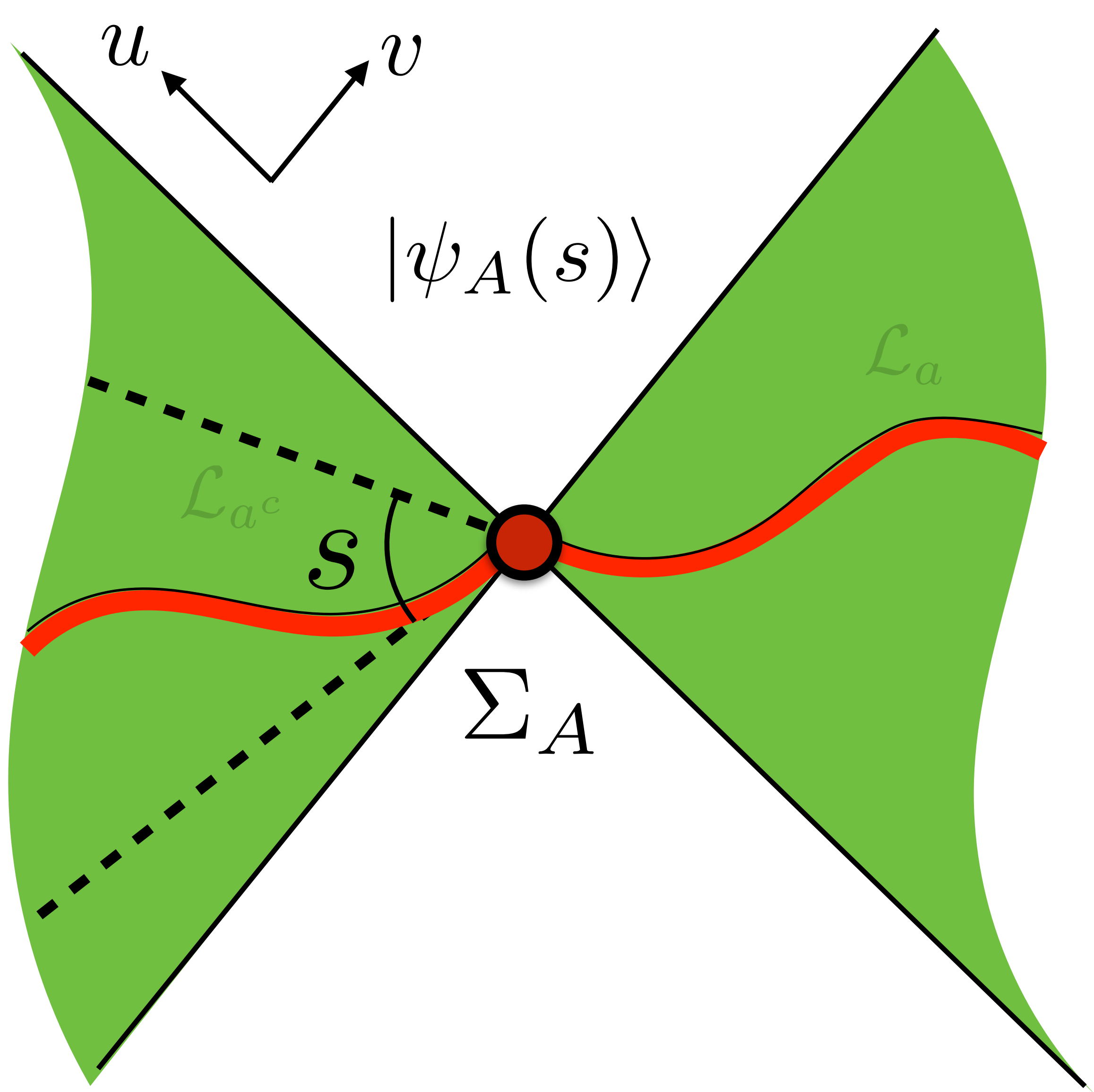

matching condition: relative boost of rapidity s across Σ_A .

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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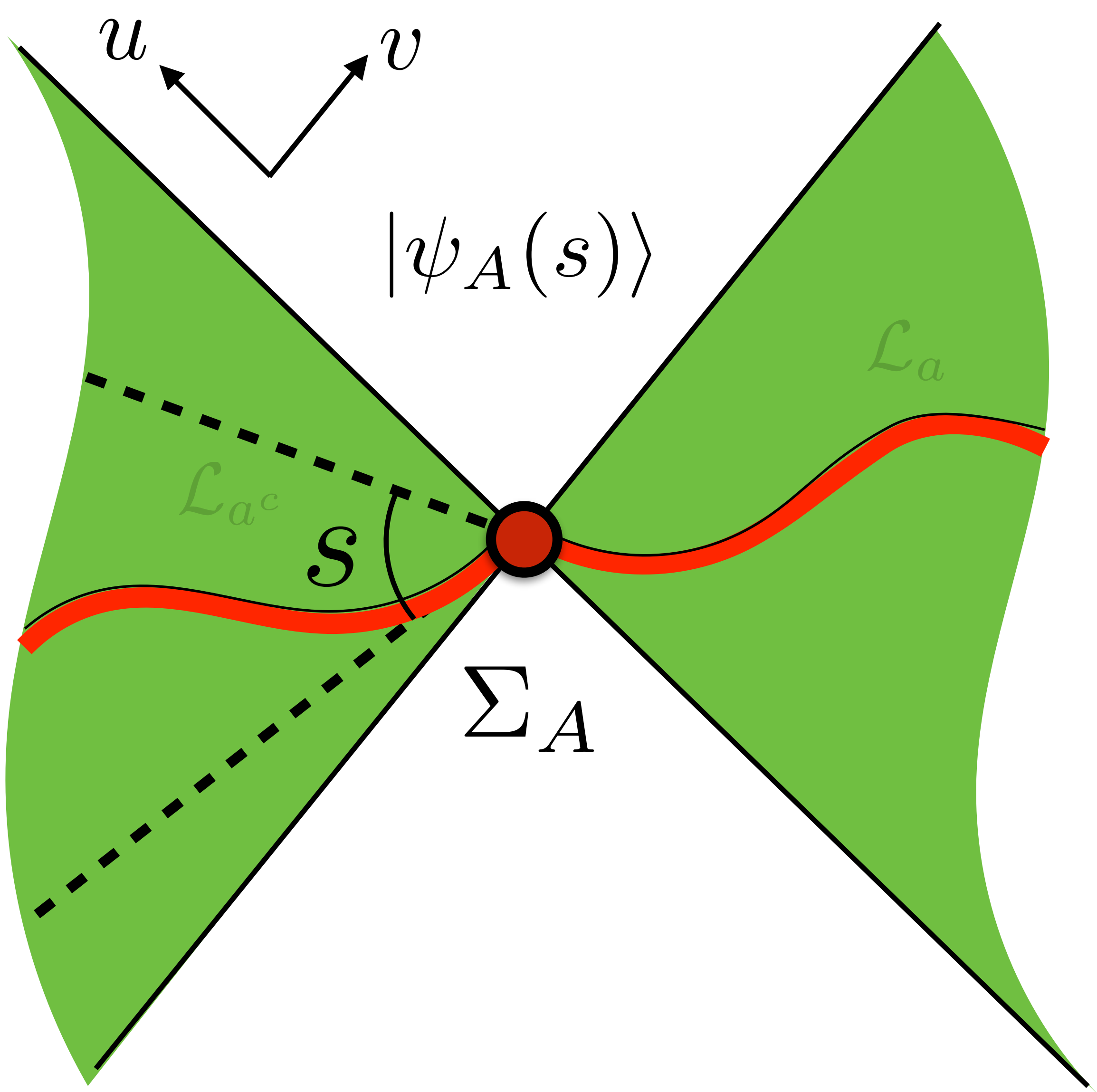

matching condition: relative boost of rapidity s across Σ_A .

modified notion of smoothness for curves across Σ_A in $|\psi_A(s)\rangle$.

Bulk modular flow in AdS/CFT

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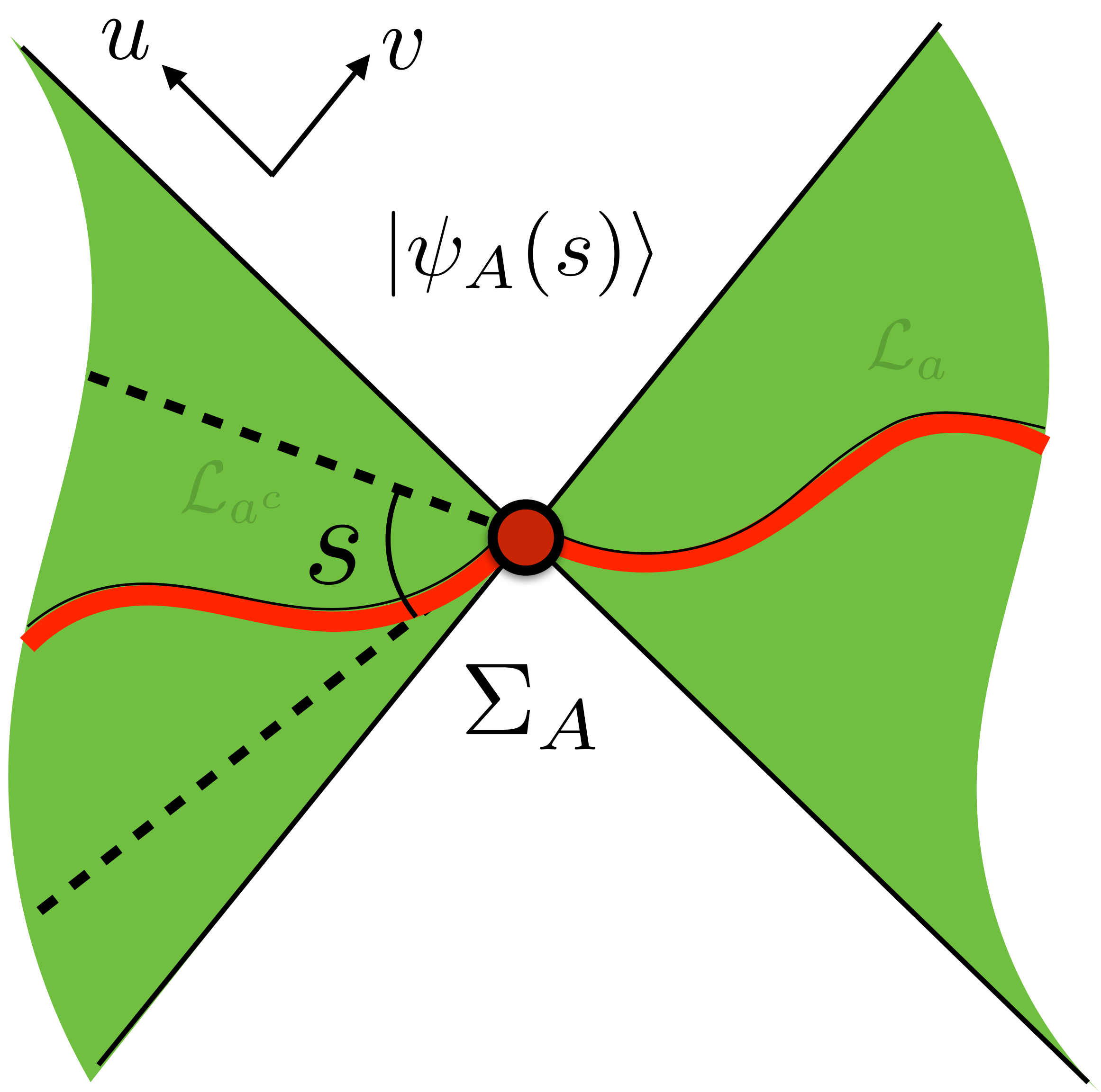

matching condition: relative boost of rapidity s across Σ_A .

modified notion of smoothness for curves across Σ_A in $|\psi_A(s)\rangle$.

fine-tuning: identify $\xi \in \Sigma_A$ s.t. at ξ

$$p_{\|}\left[\mathcal{L}(\xi, x_1)\right] = p_{\|}\left[\mathcal{L}(\xi, x_2)\right]$$

then

$$s(x_1, x_2) = \frac{1}{4\pi} \ln \left(\frac{p_u\left[\mathcal{L}(\xi, x_1)\right]}{p_v\left[\mathcal{L}(\xi, x_1)\right]} \right) \left(\frac{p_v\left[\mathcal{L}(\xi, x_2)\right]}{p_u\left[\mathcal{L}(\xi, x_2)\right]} \right)$$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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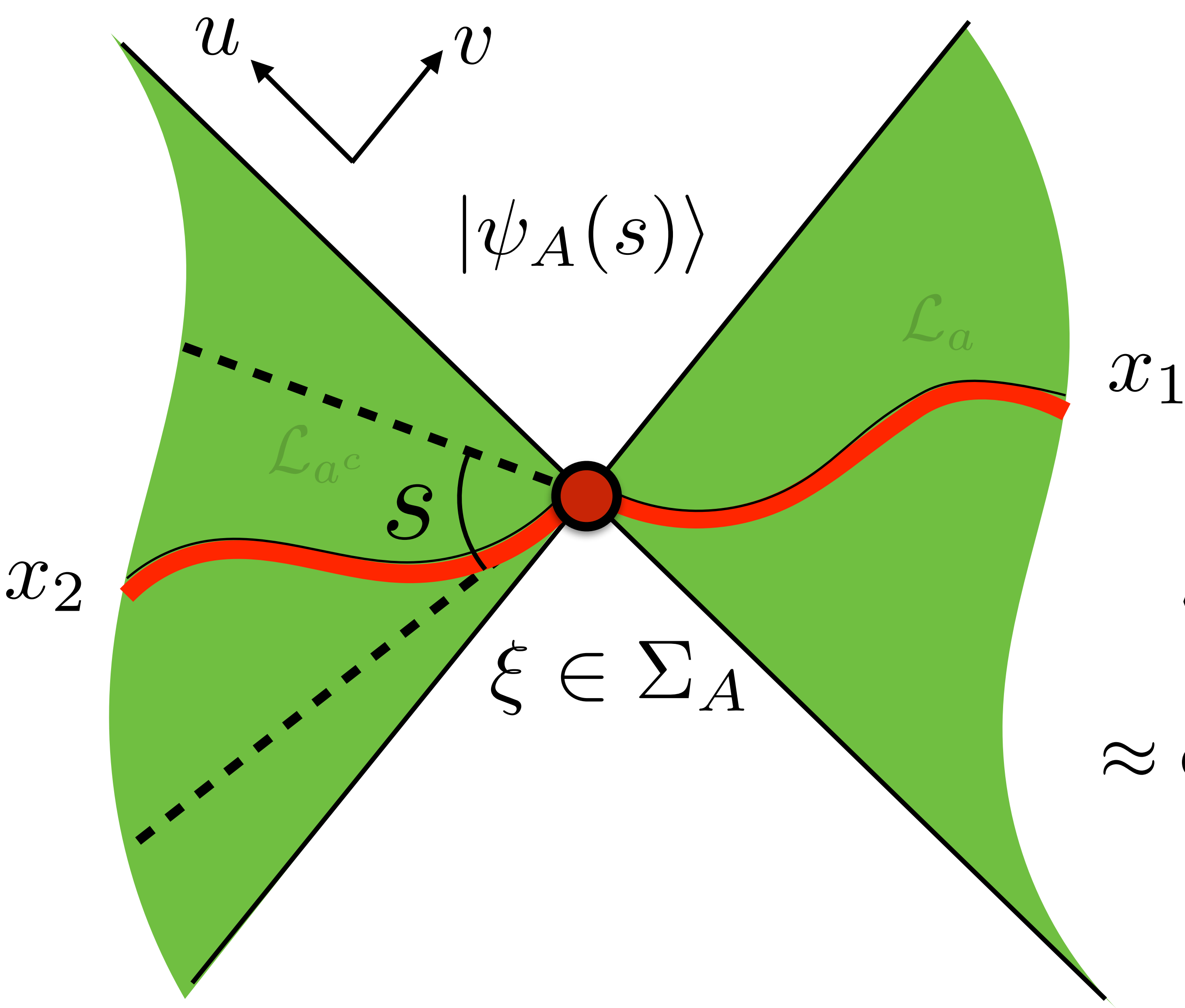


Therefore, for $s^* = s(x_1, x_2)$

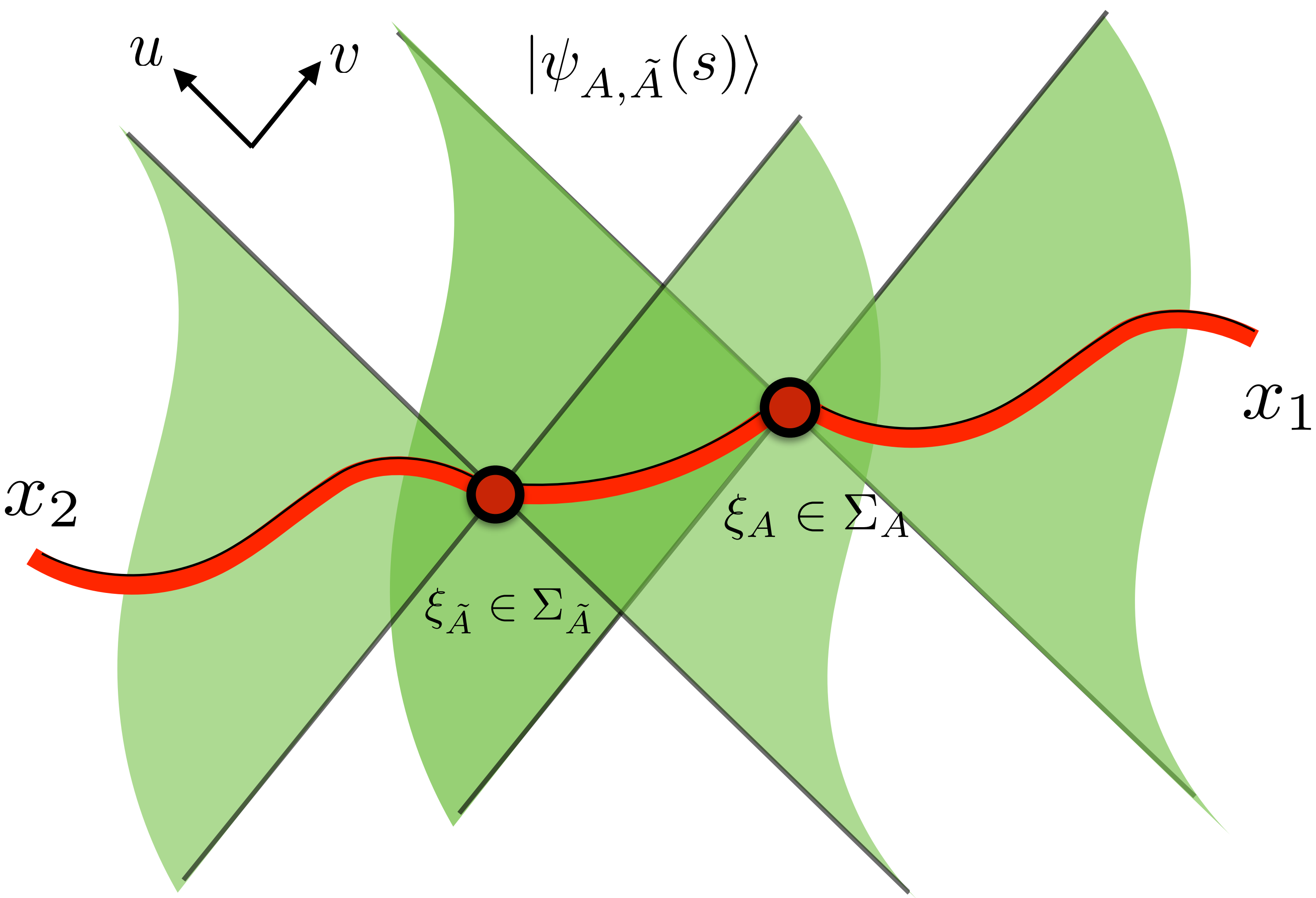
$$\langle \mathcal{O}_1 \mathcal{O}_2^A(s^*)\rangle_\psi = \langle \mathcal{O}_1 \mathcal{O}_2\rangle_{\psi_A(s^*)}$$

$$\approx \exp\left[-m\mathcal{L}(\xi, x_1) - m\mathcal{L}(\xi, x_2)\right]$$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

We can extend this to the "double modular flow": $|\psi_{A,\tilde{A}}(s)\rangle = e^{-isH^{\psi}_{\tilde{A}}} e^{isH^{\psi}_{A}} |\psi\rangle$



matching conditions at

$$\xi_A \in \Sigma_A,\ \xi_{\tilde{A}} \in \Sigma_{\tilde{A}}$$

select $s^* = s(x_1, x_2)$

$$\langle \mathcal{O}_1^A(s^*) \mathcal{O}_2^{\tilde{A}}(s^*) \rangle_\psi = \langle \mathcal{O}_1 \mathcal{O}_2 \rangle_{\psi_{A,\tilde{A}}(s^*)}$$

$$\approx \exp\left[-m\left(\mathcal{L}(\xi_A, x_1) + \mathcal{L}(\xi_{\tilde{A}}, \xi_A) + \mathcal{L}(\xi_{\tilde{A}}, x_2)\right)\right]$$

Bulk modular flow in AdS/CFT

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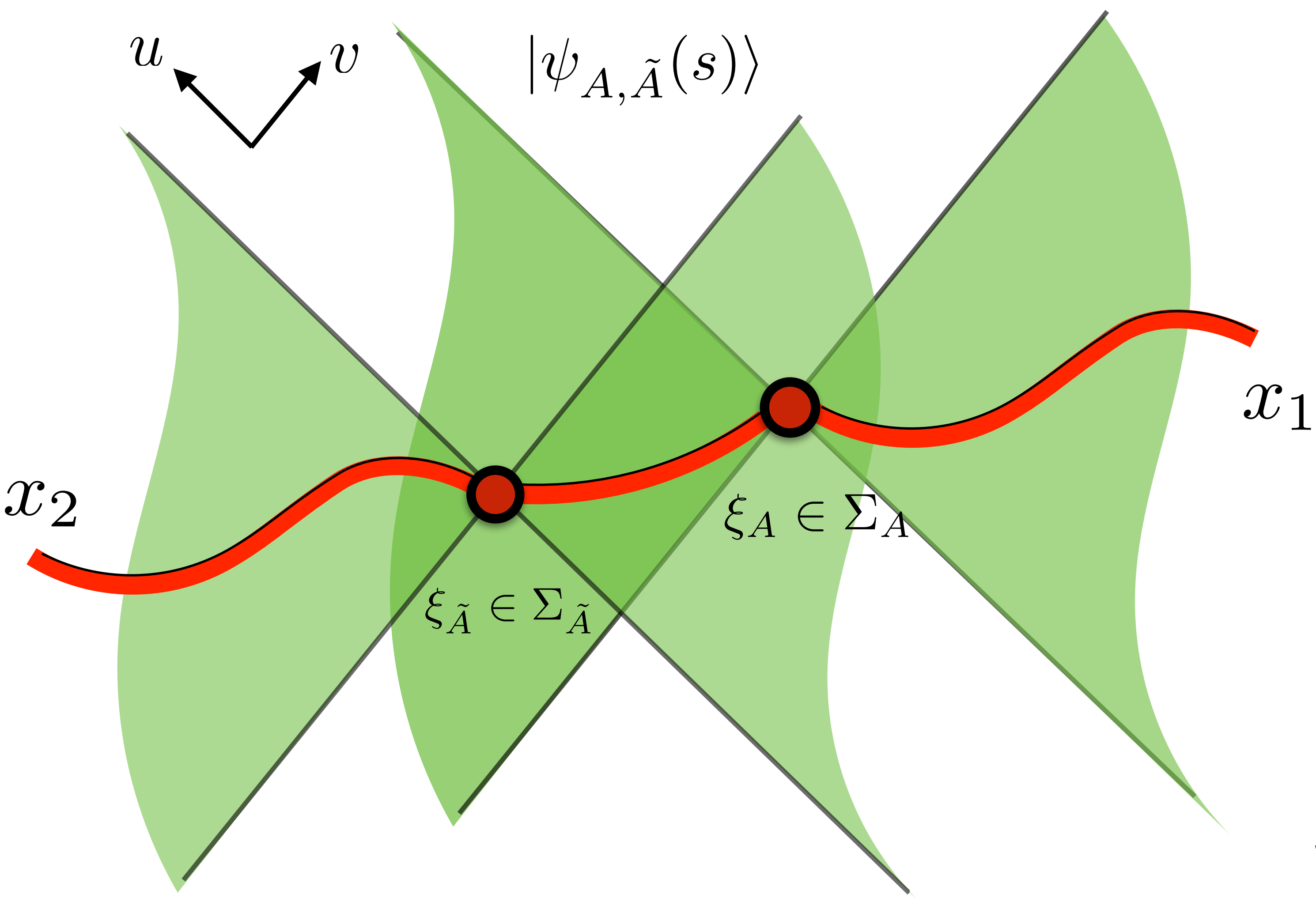


matching conditions at

$$\xi_A \in \Sigma_A, \ \xi_{\tilde{A}} \in \Sigma_{\tilde{A}}$$

select $s^* = s(x_1, x_2)$

in the near boundary limit $z \to 0$, successfully reproduced the CFT result in the light-cone limit $z \propto uv$

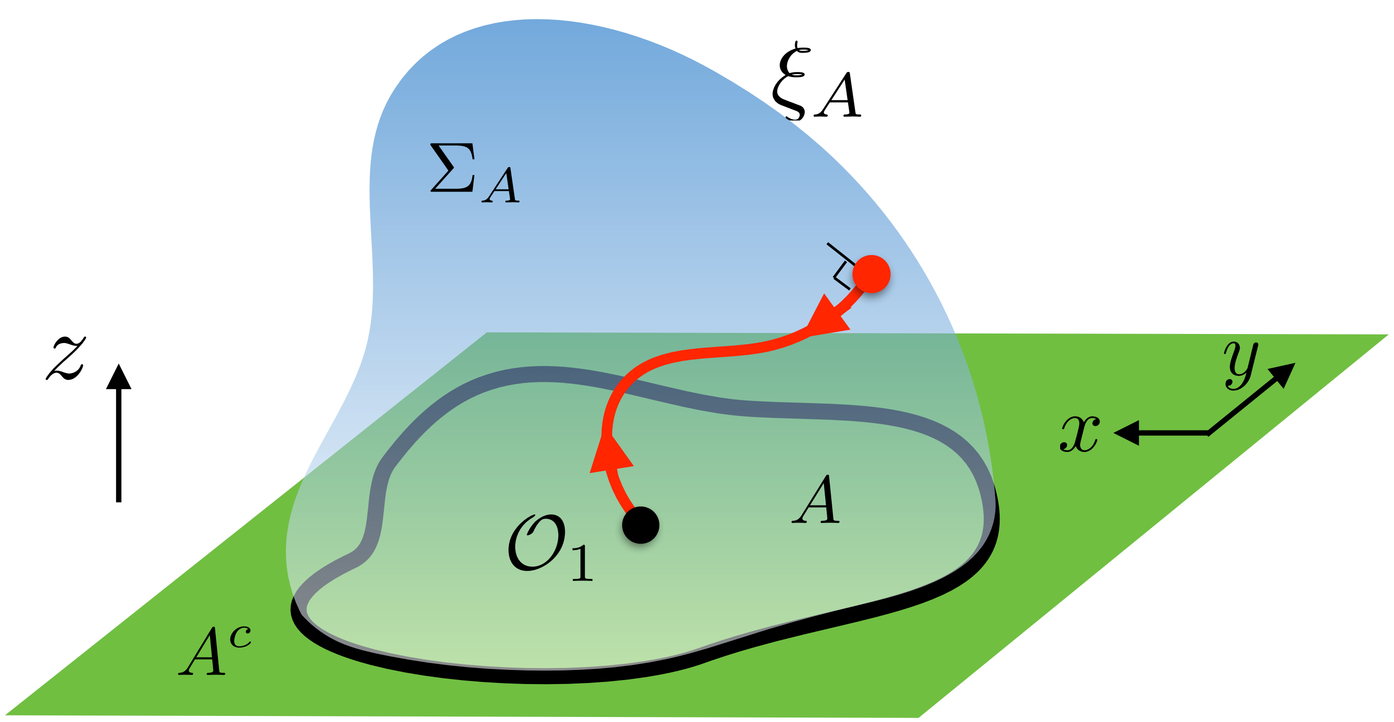
Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

Applications:

Mirror conjugation: $\mathcal{O}^J = e^{\pi K \psi_A} \mathcal{O} e^{-\pi K \psi_A} = \mathcal{O}^A(i\pi)$ K. Papadodimas, S. Raju, 2014

$f_\pi \propto \langle \mathcal{O}_1^A(i\pi)\mathcal{O}_1\rangle_\psi$ “single modular flow” with $s = i\pi$



$i\pi$ boost = reflection

$$\langle \mathcal{O}_1^J \mathcal{O}_1\rangle_\psi \approx \exp\left[-2m\mathcal{L}(\xi_A, x_1)\right]$$

Bulk modular flow in AdS/CFT

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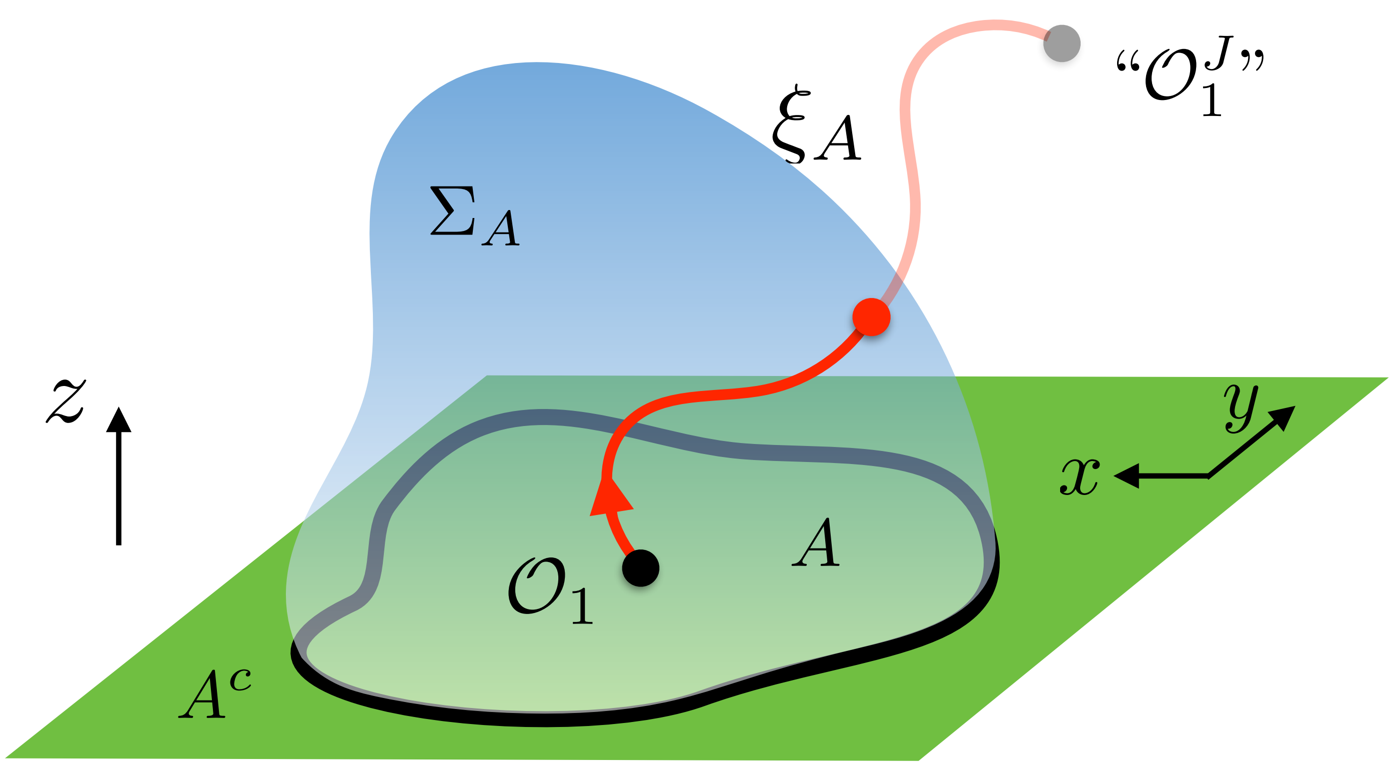

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$\langle \mathcal{O}_1^J \mathcal{O}_1\rangle_\psi \approx \exp\left[-2m\mathcal{L}(\xi_A, x_1)\right]$

RT surface serves as a mirror for implementing conjugation

Bulk modular flow in AdS/CFT

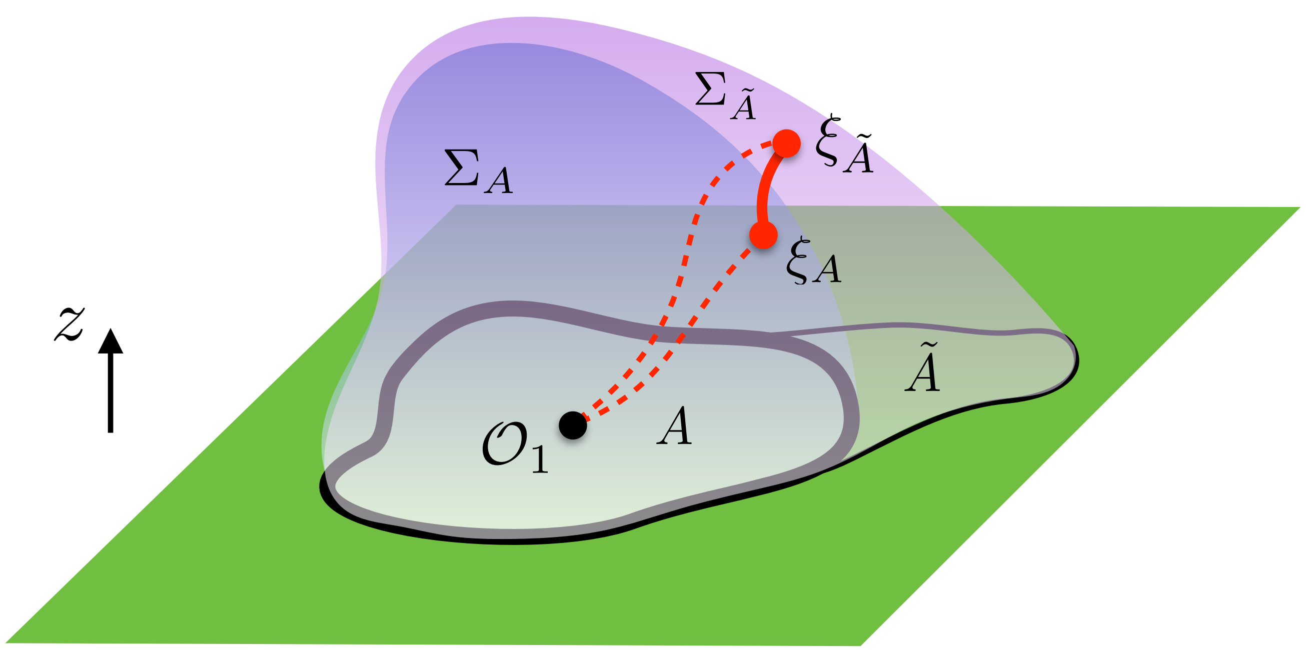
T. Faulkner, M. Li, H. Wang, 2018

Applications:

entanglement wedge nesting (EWN)

consider: $f(s) \propto \langle \mathcal{O}_1^{\tilde{A}}(s+i\pi)\mathcal{O}_1^{A}(s)\rangle_\psi$, $\tilde{A} = A + \delta A$

for $\delta A \to 0$, $f(s) = \langle \mathcal{O}_1^{J} \mathcal{O}_1 \rangle_\psi \approx \exp\left[-2m\mathcal{L}(\xi_A, x_1)\right]$ for all s



$$-m^{-1} \ln \left[\frac{\langle \mathcal{O}_1^{\tilde{A}}(s+i\pi)\mathcal{O}_1^{A}(s)\rangle_\psi}{\langle \mathcal{O}_1^{A}(i\pi)\mathcal{O}_1\rangle_\psi} \right]$$

$$\approx \mathcal{L}\left(\xi_{\tilde{A}}, \xi_A\right) + \mathcal{O}(\delta A^2)$$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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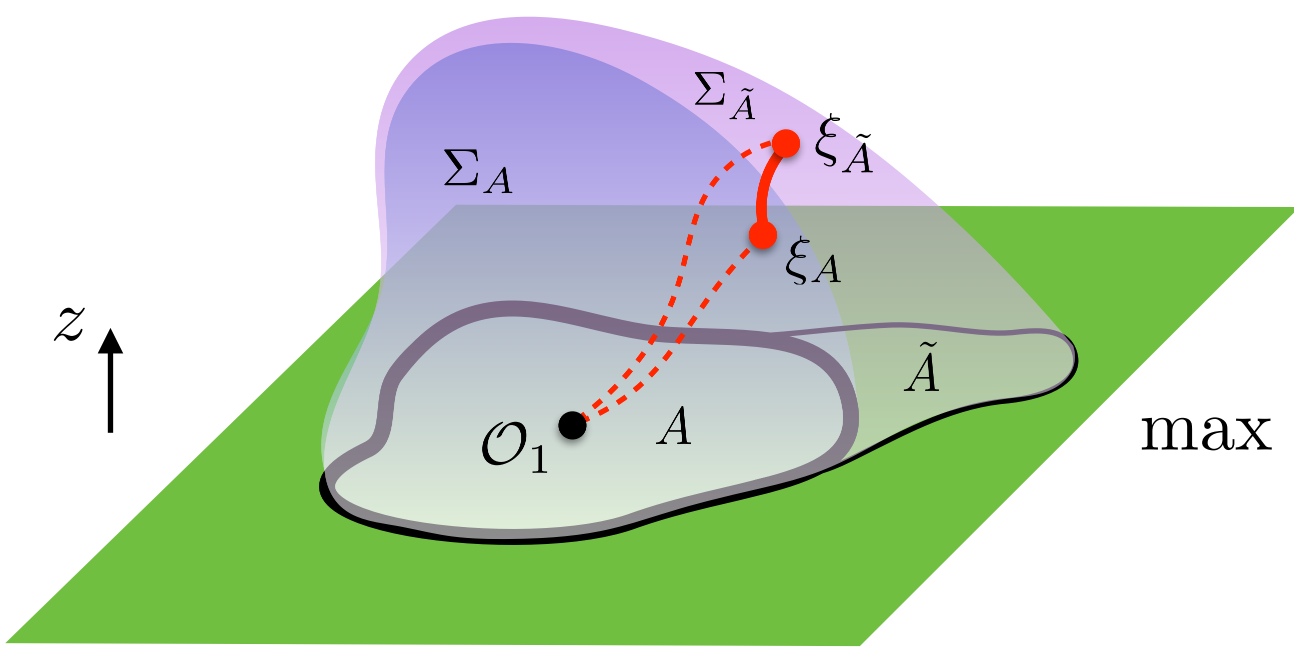

EWN in CFT (for $|\delta A| \ll |A|$)

$$\max\left\{\ln\left[\frac{\langle \mathcal{O}_1^{\tilde{A}}(s+i\pi)\mathcal{O}_1^{A}(s)\rangle_\psi}{\langle \mathcal{O}_1^{A}(i\pi)\mathcal{O}_1\rangle_\psi}\right], s \in \mathbb{R}\right\} \leq 0$$

for space-like δA

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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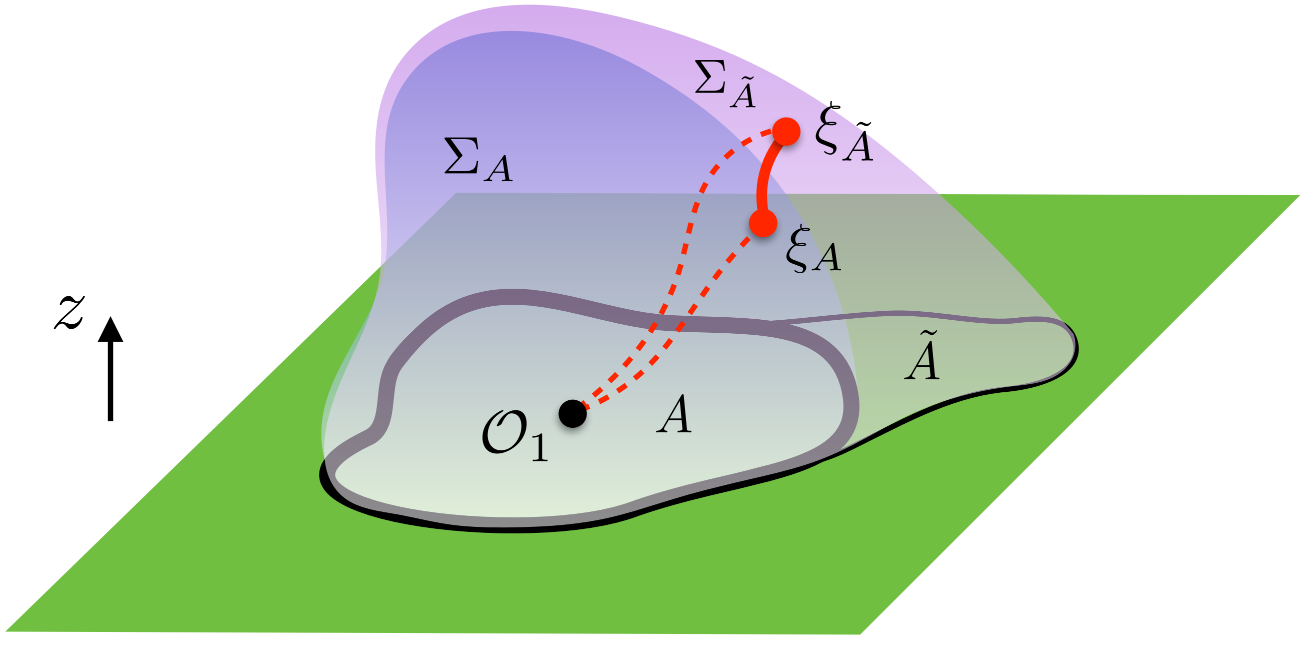

in Tomita-Takaseki theory:

can be derived from

$$|U(t)| \leq 1,\ U(t) = e^{-iK_{\tilde{A}}^{\psi}t}e^{iK_{A}^{\psi}t}$$

Conclusion/Outlook

- general proofs of energy conditions in QFTs
- physical picture encoded in the entanglement structures (modular flow)
- holographic proof of QNEC using EWN: RT surface dynamics
- boundary modular flow “knows” about these…
- prescription for (fine-tuned classes of) modular flows in AdS/CFT

Conclusion/Outlook

Future directions:

- what happens in the “Milne wedges”?
- 1/N corrections to the prescription
- other bulk constraints from boundary modular flow, e.g. quantum focusing conjecture (QFC)?

Thank you!