Large AdS black holes from QFT

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Black Hole

- Classically, absolute zero temperature due to strong attraction force.
- Proper thermodynamic system with finite temperature and entropy:

$$T = \frac{\kappa}{2\pi}$$ Hawking temperature

$$S_{\mathrm{BH}} = \frac{A}{4G_N}$$ Bekenstein-Hawking entropy

- Thermodynamic entropy has statistical, microscopic interpretation.
- Boltzmann interpretation of Bekenstein-Hawking entropy?

$$\frac{A}{4G_N} = \log \Omega_{\mathrm{micro}}$$

  - Yes, for many black hole solutions engineered in string theory.
    [Strominger, Vafa][Breckenridge, Myers, Peet, Vafa][Dijkgraaf, Verlinde, Verlinde] …

- In this talk, I want to answer the question for some AdS black holes.
  [Benini, Hristov, Zaffaroni][Azzurli et al.][Hosseini, Hristov, Passias][Hosseini, Yaakov, Zaffaroni] …

AdS-Schwarzschild Black Hole

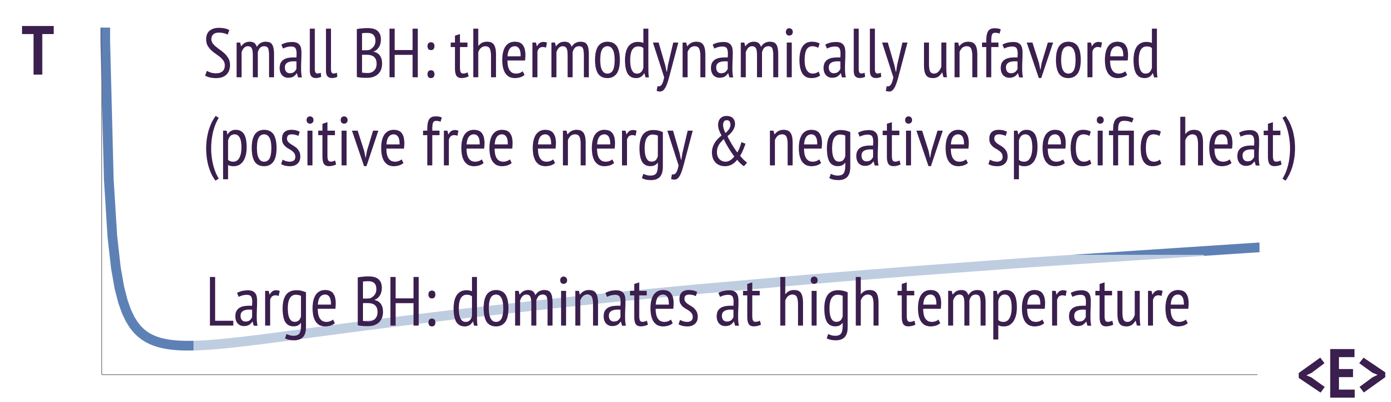
- Example: AdS-Schwarzschild solution. [Hawking, Page]

$$ds^2 = \left(\frac{r^2}{b^2} + 1 - \frac{w_2 M}{r^2}\right) dt^2 + \frac{dr^2}{\left(\frac{r^2}{b^2} + 1 - \frac{w_2 M}{r^2}\right)} + r^2 d\Omega_3^2$$

- Smooth and complete iff the ***t***-coordinate has the periodicity:

$$t \sim t + \beta \quad \text{with} \quad \beta^{-1} = \frac{r_+}{\pi b^2} + \frac{1}{2\pi r_+}\,, \;\; r_+^2 = -\frac{b^2}{2} + \frac{b}{2}\sqrt{b^2 + 4 w_2 M}$$

- Two distinct branches:

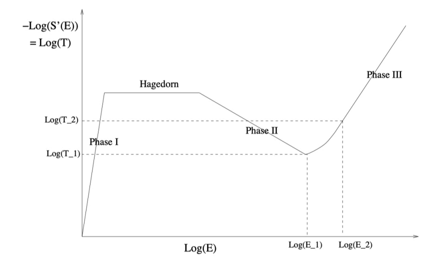


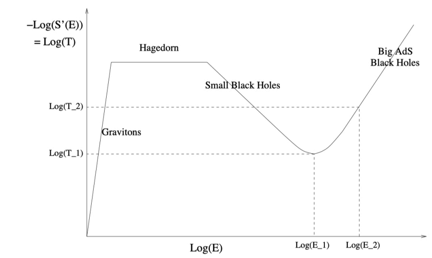
- Hawking-Page transition
  - $T < T_{HP}$: thermal graviton in the global AdS.
  - $T > T_{HP}$: large AdS-Schwarzschild black hole. $F \sim 1/G_N \sim N^2$.

Deconfinement Phase Transition

- In the large N gauge theory, the Hawking-Page transition is realized as the deconfinement phase transition. [Witten '98]
- Even in the weakly coupled regime of some large N gauge theory, the phase diagram looks analogous to that of AdS black hole.

[Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk]





- This talk: 1/16 BPS black hole in $AdS_{5(7)}$.

AdS_5 [Gutowski, Reall] [Kunduri, Lucietti, Reall] [Chong, Cvetic, Lu, Pope] [Wu], AdS_7 [Chong, Cvetic, Lu, Pope] [Chow]

  - $U(1)^2 \subset SO(4)$ angular momenta, $U(1)^3 \subset SO(6)$ electric charges.
  - Preserves 2 supercharges.

$$\mathcal{Q}^{+++}_{--}, \mathcal{S}^{---}_{++} \quad \longrightarrow \quad E = J_1 + J_2 + Q_1 + Q_2 + Q_3$$

  - Exhibits $O(N^2)$ entropy at $O(N^2)$ angular momenta and charges.

$$S = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \tfrac{1}{2}N^2(J_1+J_2)} \sim \mathcal{O}(N^2)$$

Plan

- Introduction

- Large AdS_5 black holes from N=4 SYM

- Deconfinement phase transition in N=4 SYM

- Conclusion

Superconformal Index

- Enumerate all BPS microstates preserving supercharges $\mathcal{Q}^{+++}_{--}, \mathcal{S}^{---}_{++}$
- Index of N=4 Yang-Mills on S^3 X R^1 : [Romelsberger] [Kinney, Maldacena, Minwalla, Raju]

$$Z = \mathrm{Tr}\Big[e^{-\beta\mathcal{E}} \prod_{I=1}^{3} e^{-\Delta_I Q_I} \prod_{i=1}^{2} e^{-\omega_i J_i}\Big] \qquad \text{with} \qquad \mathcal{E} = \big\{\mathcal{Q}^{+++}_{--}, \mathcal{S}^{---}_{++}\big\}$$

- Chemical potentials for SO(4) isometry and SO(6) R-symmetry.

$$\sum_{I=1}^{3} \Delta_I = \sum_{i=1}^{2} \omega_i + 2\pi i \longrightarrow \begin{array}{l} \big\{e^{-\Delta_I Q_I - \omega_i J_i}, \mathcal{Q}^{+++}_{--}\big\} = 0 \\ \big\{e^{-\Delta_I Q_I - \omega_i J_i}, \mathcal{S}^{---}_{++}\big\} = 0 \end{array}$$

- Independent of the regulator β, so formally take β → 0.
- SUSY protection. Can be evaluated from the free QFT calculus.

$$Z = \oint [d\alpha] \cdot \exp\Big[\sum_{a,b=1}^{N} \sum_{n=1}^{\infty} \frac{1}{n}\Big(1 + \sum_{s_1,s_2,s_3=\pm 1} \frac{s_1 s_2 s_3 (-1)^{n-1} e^{\frac{n s_I \Delta_I}{2}}}{2\sinh\frac{n\omega_1}{2} \cdot 2\sinh\frac{n\omega_2}{2}}\Big) e^{in\alpha_{ab}}\Big]$$

- Agrees with the BPS graviton index at low “temperature”.

Asymptotic Free Energy

- Does the index see the deconfinement at high "temperature"?
- Consider the asymptotic free energy in the Cardy limit $|\omega_{1,2}| \ll 1$ where large angular momentum ("energy") states dominate.

- Saddle point approximation: $\alpha_1 = \cdots = \alpha_N$ is most important.

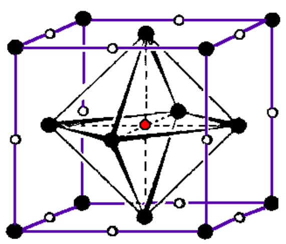
$$\log Z = -\frac{N^2}{\omega_1\omega_2} \sum_{s_1 s_2 s_3 = +1} \left[\mathrm{Li}_3\left(-e^{\frac{s_I \Delta_I}{2}}\right) - \mathrm{Li}_3\left(-e^{-\frac{s_I \Delta_I}{2}}\right) \right]$$

Use tri-logarithm identity

$$\mathrm{Li}_3(-e^{x}) - \mathrm{Li}_3(-e^{-x}) = -\frac{x^3}{6} - \frac{\pi^2 x}{6}$$

in the "canonical chamber"

$$-2\pi < \mathrm{Im}(+\Delta_1 + \Delta_2 + \Delta_3) < 2\pi, \quad -2\pi < \mathrm{Im}(+\Delta_1 - \Delta_2 - \Delta_3) < 2\pi$$
$$-2\pi < \mathrm{Im}(-\Delta_1 + \Delta_2 - \Delta_3) < 2\pi, \quad -2\pi < \mathrm{Im}(-\Delta_1 - \Delta_2 + \Delta_3) < 2\pi$$



$$\log Z = \frac{N^2 \Delta_1 \Delta_2 \Delta_3}{2\omega_1\omega_2} \quad \text{with} \quad \textstyle\sum_{I=1}^{3} \Delta_I = \sum_{i=1}^{2} \omega_i + 2\pi i$$

- Respects the 4πi periodicity of chemical potentials.

Asymptotic Free Energy

- Alternative derivation: the background field action in $\beta/r \ll \omega_{1,2} \ll 1$
  - β corresponds to the circumference of the temporal circle.
  - S^3 effective action of background fields. [Di Pietro, Komargodski]

$$ds_4^2 = r^2 \left[d\theta^2 + \sum_i n_i^2 d\phi_i^2 + \frac{r^2(\sum_i \omega_i n_i^2 d\phi_i)^2}{\beta^2(1 - r^2 \sum_i \frac{n_i^2 \omega_i^2}{\beta^2})} \right] + e^{-2\Phi}(d\tau + a)^2$$ metric

$$\mathcal{A}^I = -A_4^I a$$ gauge field

$$a = -i \frac{r^2 \sum_i \omega_i n_i^2 d\phi_i}{\beta(1 - \sum_i r^2 \frac{n_i^2 \omega_i^2}{\beta^2})}$$ graviphoton

$$e^{-2\Phi} = 1 - r^2 \sum_i \frac{n_i^2 \omega_i^2}{\beta^2}$$ dilaton

$$A_4^I = -\frac{i\Delta^I}{\beta}$$ scalar

- Most terms are suppressed. Only contribution comes from

$$S_{\mathrm{CS}} = -\frac{iN^2}{8\pi} \cdot \frac{\beta}{2\pi} \int_{S^3} C_{IJK} \left(A_4^I \mathcal{A}^J \wedge d\mathcal{A}^K + A_4^I A_4^J \mathcal{A}^K \wedge da + \frac{1}{3} A_4^I A_4^J A_4^K a \wedge da \right) = -\frac{N^2 \Delta_1 \Delta_2 \Delta_3}{2\omega_1 \omega_2}$$

  - Gauge non-invariant CS terms: needed to match ’t Hooft anomaly
  - Generalizable to non-Lagrangian QFTs, e.g., 6d (2,0) SCFT.

Large AdS Black Hole

- Having obtained the large N superconformal index, take an inverse Laplace transformation to extract the degeneracy at given charges.

$$\Omega(Q_I, J_i) = \int d\Delta_I \, d\omega_i \;\; Z(\Delta_I, \omega_i) \exp\Big(\textstyle\sum_I \Delta_I Q_I + \sum_i \omega_i J_i \Big)$$

- Asymptotic degeneracy at macroscopic charges can be evaluated by the saddle point approximation, i.e., the Legendre transformation of the asymptotic free energy in the Cardy limit. [Hosseini, Hristov, Zaffaroni]

$$S(\Delta_I, \omega_i) = +\frac{N^2 \Delta_1 \Delta_2 \Delta_3}{2\omega_1 \omega_2} + \sum_{I=1}^{3} \Delta_I Q_I + \sum_{i=1}^{2} \omega_i J_i \quad \text{with} \quad \textstyle\sum_{I=1}^{3} \Delta_I - \sum_{i=1}^{2} \omega_i = 2\pi i$$

$$S(\Delta_I, \omega_i) = -\frac{N^3 (\Delta_1 \Delta_2)^2}{24 \omega_1 \omega_2 \omega_3} + \sum_{I=1}^{2} \Delta_I Q_I + \sum_{i=1}^{3} \omega_i J_i \quad \text{with} \quad \textstyle\sum_{I=1}^{2} \Delta_I - \sum_{i=1}^{3} \omega_i = 2\pi i$$

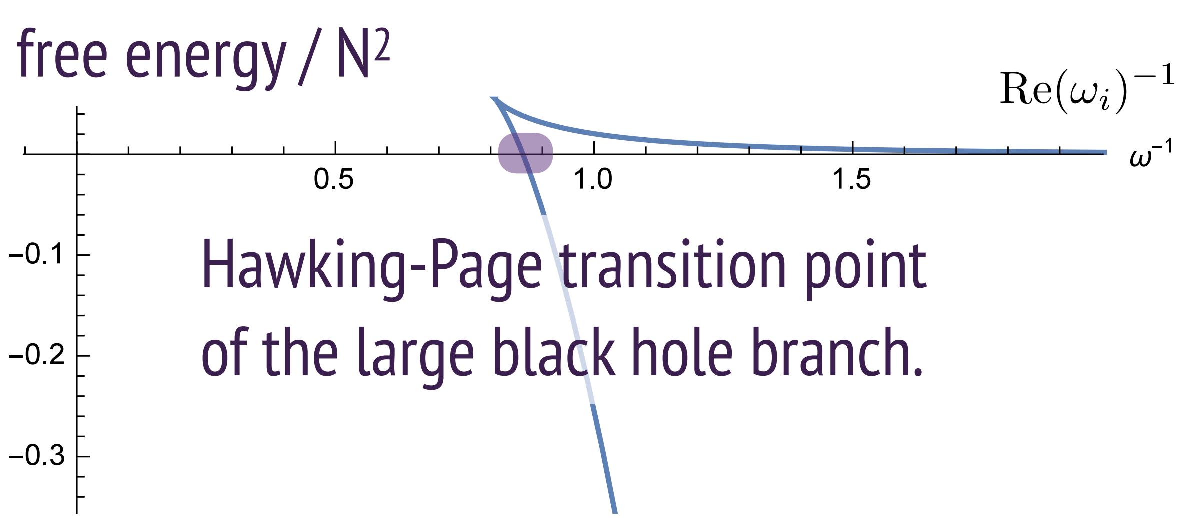
Large AdS Black Hole

- Further imposing the charge relation of known BPS black holes, it matches the known entropy formula of BPS black holes!

$$\text{AdS}_5: \; S = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \tfrac{1}{2}N^2(J_1+J_2)}$$

$$\text{AdS}_7: \; S = 2\pi\sqrt{\frac{3(Q_1^2Q_2 + Q_1Q_2^2) - N^3(J_1J_2 + J_2J_3 + J_3J_1)}{3(Q_1+Q_2) - N^3}}$$

- Asymptotic free energy = the free energy of the known BH solution.



Small BH: thermodynamically unfavored (positive free energy, negative specific heat)

Large BH: dominates at high "temperature"

Analogous to AdS-Schwarzschild solution.

Plan

- Introduction
- Large AdS_5 black holes from N=4 SYM
- Deconfinement phase transition in N=4 SYM
- Conclusion

Complex Fugacity

- Superconformal index clearly sees the deconfining phase at high "temperature," in contrast to what had been commonly believed:

  Massive boson/fermion cancelation prevents the index from seeing the $O(N^2)$ free energy of deconfining phase.

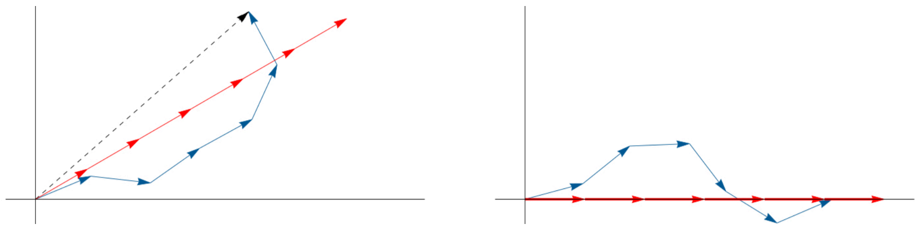
- B/F cancelation reduces the asymptotic degeneracy at two levels.
  - Case 1. the actual microstate degeneracy can be $O(N^0)$.
  - Case 2. the degeneracy grows quickly but with alternating signs.

$$
\begin{aligned}
&1 + 3x^2 - 2x^3 + 9x^4 - 6x^5 + 11x^6 - 6x^7 + 9x^8 + 14x^9 - 21x^{10} + 36x^{11} - 17x^{12} - 18x^{13} + 114x^{14} - 194x^{15} \\
&+ 258x^{16} - 168x^{17} - 112x^{18} + 630x^{19} - 1089x^{20} + 1130x^{21} - 273x^{22} - 1632x^{23} + 4104x^{24} - 5364x^{25} + 3426x^2 \\
&+ 3152x^{27} - 13233x^{28} + 21336x^{29} - 18319x^{30} - 2994x^{31} + 40752x^{32} - 76884x^{33} + 78012x^{34} - 11808x^{35} + \cdots
\end{aligned}
$$

  Macroscopic charge approximation is insensitive to the change by 1 or 2 charge units. Nearby terms are effectively smeared out. The asymptotic degeneracy becomes $O(N^0)$ after "cancelation".

Complex Fugacity

- The macroscopic B/F cancelation can be obstructed by introducing (relative) phase factors between nearby terms.



- Legendre transformation chooses an optimal value to maximize the asymptotic degeneracy.

$$\Delta_1 = \Delta_2 = \Delta_3 \quad \text{and} \quad \omega_1 = \omega_2 \ll 1 \quad \longrightarrow \quad \Delta_1 = \Delta_2 = \Delta_3 = \tfrac{2}{3}\pi i$$

- Allowing the chemical potentials to be complex-valued, I want to trace how the deconfinement phase transition happens in the index.

[Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk] [Kinney, Maldacena, Minwalla, Raju]

Confining Saddle Point

- Introducing the large N eigenvalue distribution, the index becomes

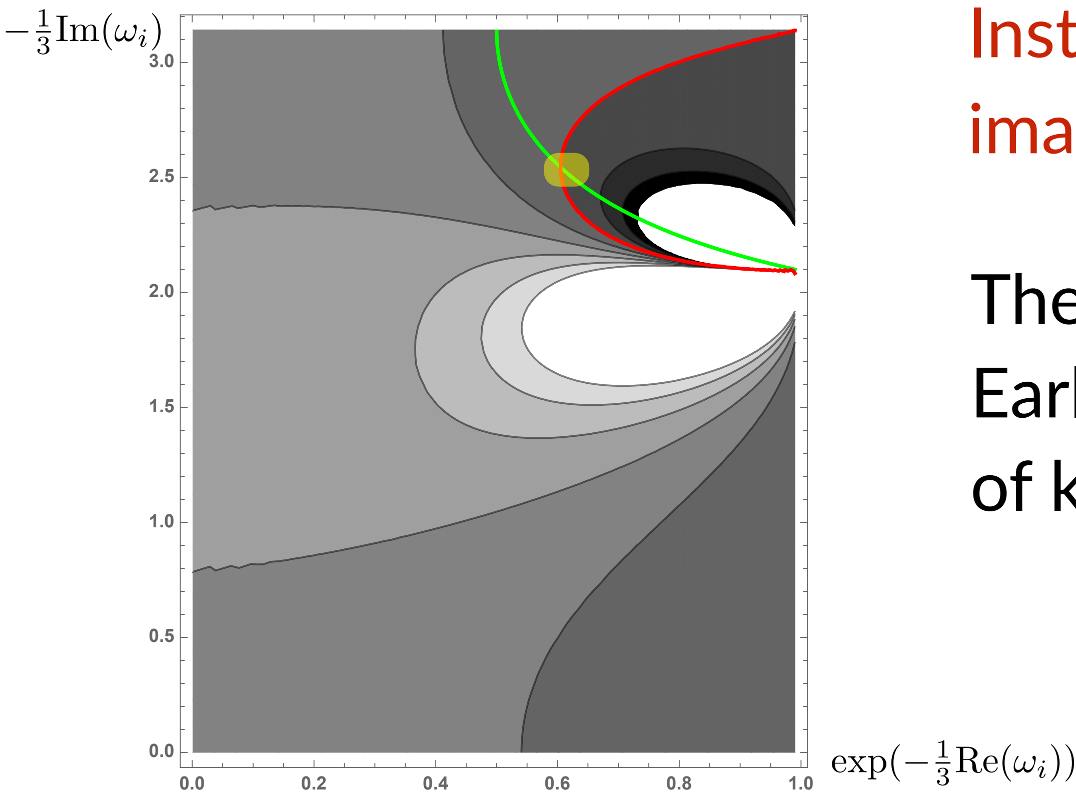
$$Z = \int \prod_{n\neq 0} d\rho_n \, \exp\left(-\sum_{n=1}^{\infty} \frac{N^2}{n} \frac{\prod_I (1-e^{-n\Delta_I})}{\prod_i (1-e^{-n\omega_i})} \rho_n \rho_{-n}\right) \quad \text{where} \quad \rho_n = \frac{1}{2\pi N} \sum_{a=1}^{N} e^{-in\alpha_a}$$

  - Polyakov loop: an order parameter for (de)confinement transition.

- Confining saddle point: $\rho_n = 0$
  - Most dominant saddle point at *real-valued* chemical potentials.
  - Gaussian integral = the supergraviton spectrum on $AdS_5 \times S^5$.
  - Becomes unstable if $\mathrm{Re}\left[\frac{\prod_I (1-e^{-\Delta_I})}{\prod_i (1-e^{-\omega_i})}\right] < 0$ with complex Δ_I, ω_i.

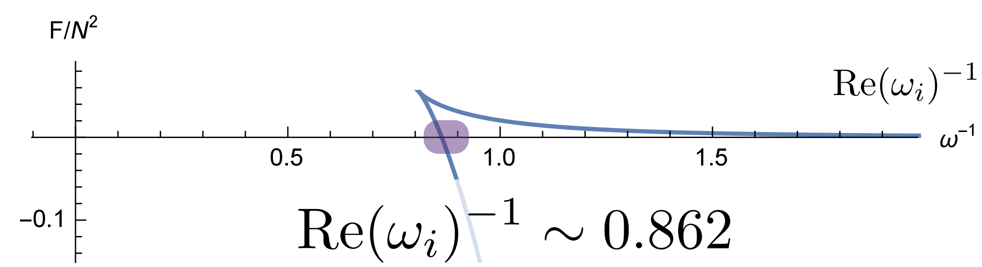
Confining Saddle Point

- At equal charges and angular momenta, $\Delta_1 = \Delta_2 = \Delta_3$ and $\omega_1 = \omega_2$ the confining saddle point becomes unstable on the **red** line.



Instability arises only at suitably chosen imaginary value of chemical potentials.

The earliest instability point: $\mathrm{Re}(\omega_i)^{-1} \sim 0.663$
Earlier than the Hawking-Page transition point of known AdS$_5$ black hole



- Since true deconfinement should happen before/at the instability point, this implies the existence of new black hole saddle point!

Conclusion

- Superconformal index sees the BPS black holes in $AdS_{5/7}$.
- The microstate degeneracy of known BPS black hole solution was obtained from the superconformal index at the Cardy limit.

- *Imaginary* chemical potentials obstructs the massive B/F cancelation.
- Confining saddle point, dominant at low “temperature,” becomes unstable at high “temperature” with *imaginary* chemical potentials.

- The earliest deconfining saddle point is *not* of the known black hole, thereby predicting a new BPS black hole in asymptotic AdS_5.

- Many other saddle points showing $O(N^2)$ degeneracy was found. Can we identify them with some hairy BPS black hole solutions?