Higher Spin and Yangian

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Sanya, 2019/01/07

Reference

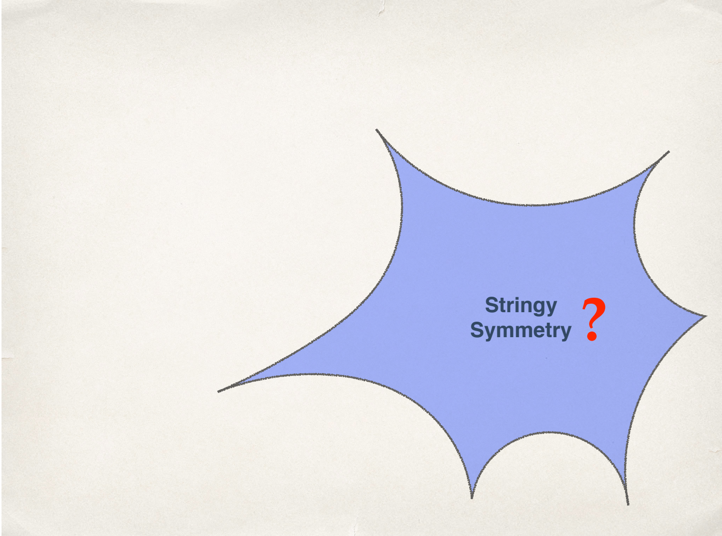
1. **Higher Spins and Yangian Symmetries**
   JHEP **1704**, 152 (2017), [arXiv:1702.05100]
   with *Matthias Gaberdiel, Rajesh Gopakumar, and Cheng Peng*

2. **Twisted sectors from plane partitions**
   JHEP **1609**, 138 (2016), [arXiv:1606.07070]
   with *Shouvik Datta, Matthias Gaberdiel, and Cheng Peng*

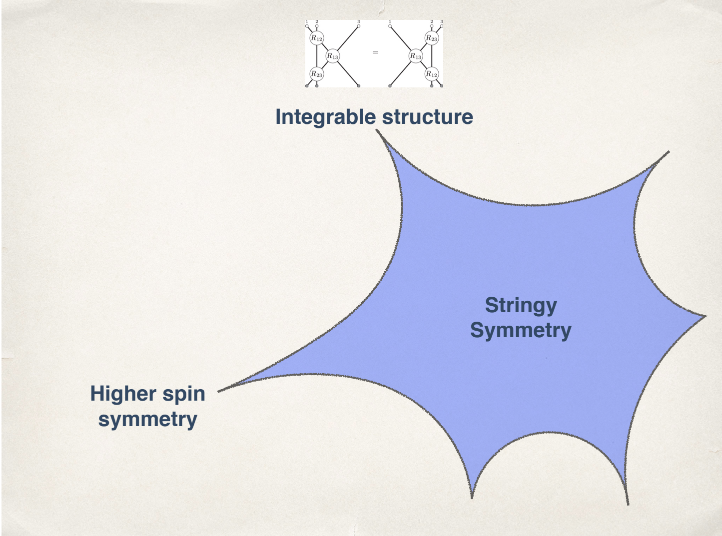
3. **The supersymmetric affine yangian**
   JHEP **1805**, 200 (2018), [arXiv:1711.07449]
   with *Matthias Gaberdiel, Cheng Peng, and Hong Zhang*

4. **Twin plane partitions and $\mathcal{N}=2$ affine yangian**
   JHEP **1811**, 192 (2018), [arXiv:1807.11304]
   with *Matthias Gaberdiel and Cheng Peng*

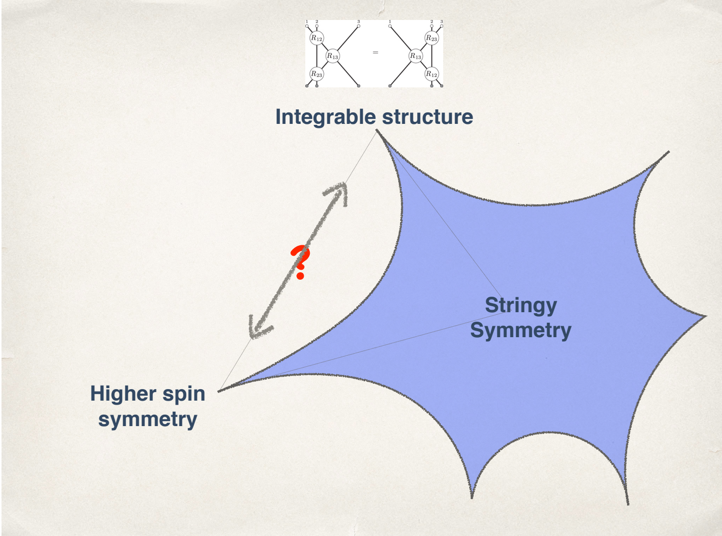
There is a large hidden symmetry in string theory



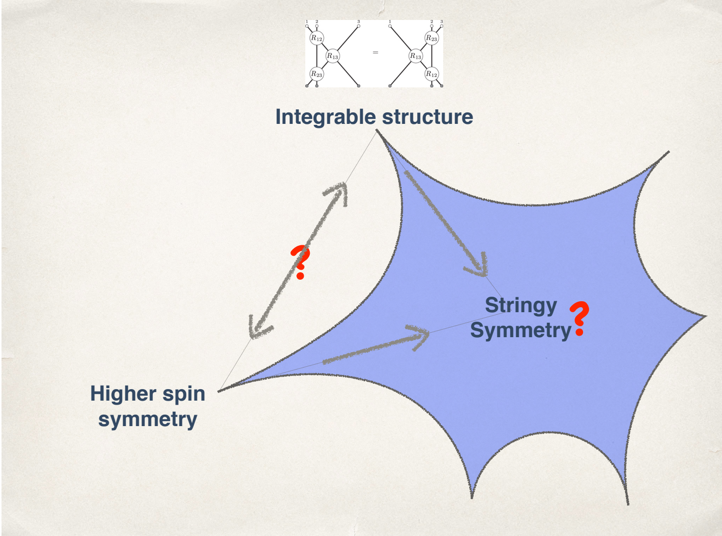
Different manifestation of stringy symmetry



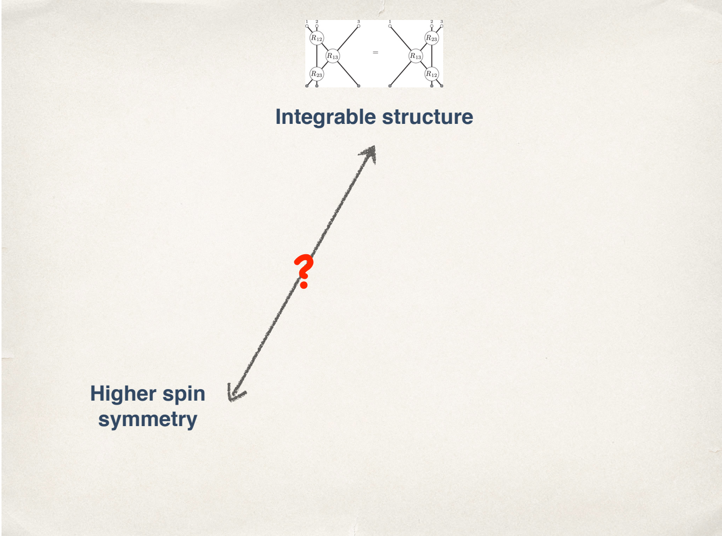
Different manifestation of stringy symmetry



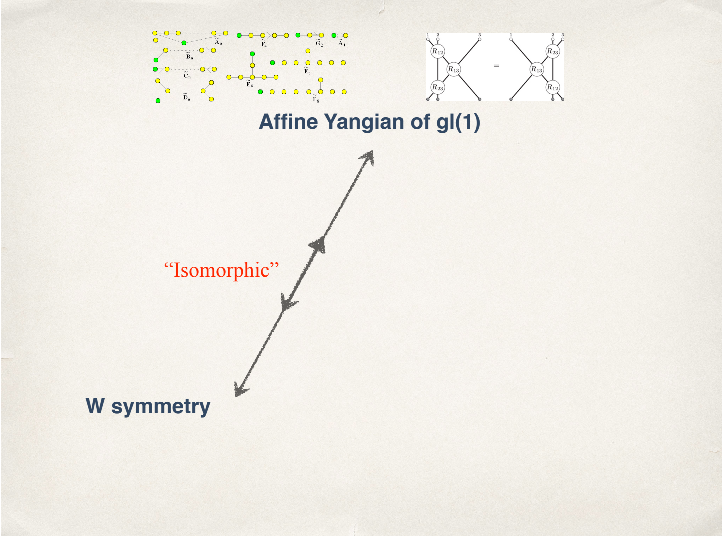
Different manifestation of stringy symmetry



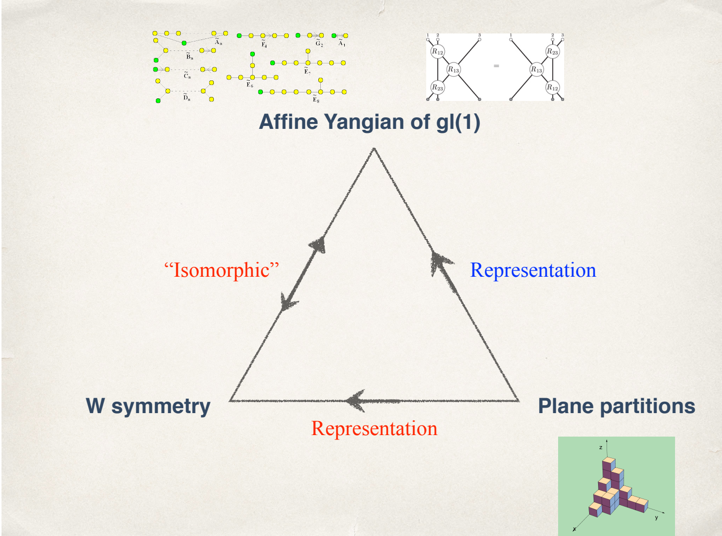
Today



A concrete relation between HS and integrability



Application: plane partition as representations of $\mathcal{W}_\infty$



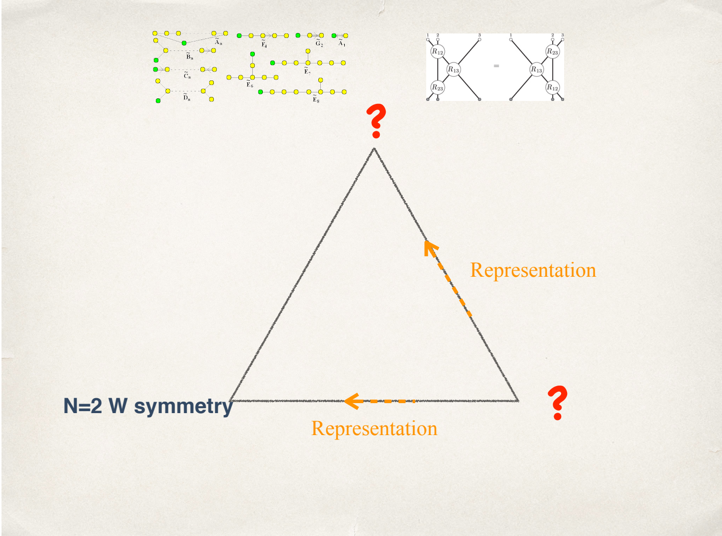
Two questions

1. Supersymmetrize $\triangle$?
2. $\triangle$ as lego pieces for new VOA/affine Yangian?

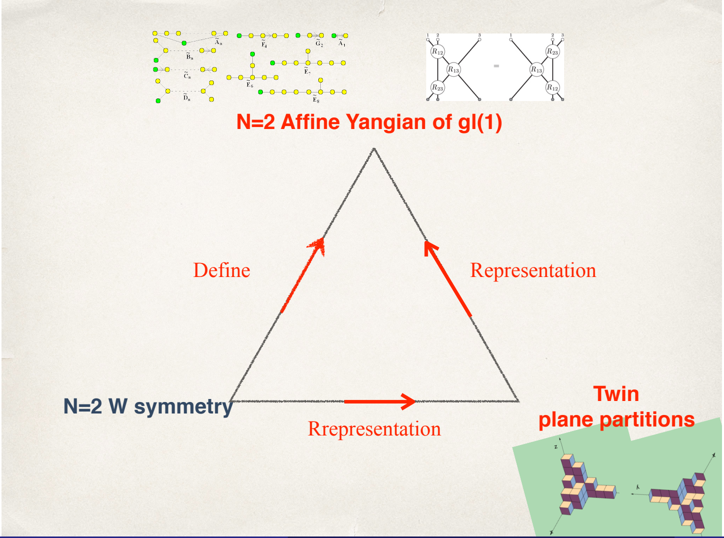
A surprising (partial) answer

Glue two $\triangle$ to get $\mathcal{N}=2$ version of $\triangle$

$\mathcal{N} = 2$ version?



New Yangian algebra from W algebra



Finite truncation of affine Yangian of $\mathfrak{gl}_1$

Fukuda Matsuo Nakamura Zhu '15

Prochazka '15

- gives chiral algebra of Y-junction

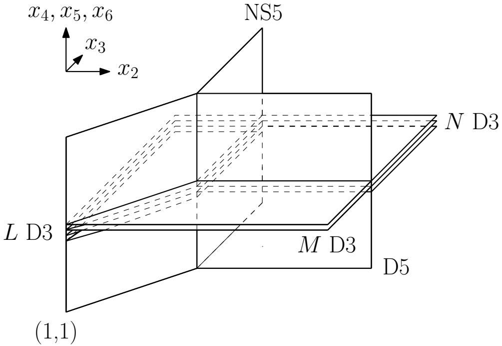
Gaiotto Rapcak '17

- Gluing of these finite truncations should give chiral algebra of Y-junction webs

Rapcak Prochazka'17

5-brane junction with D3 brane interfaces

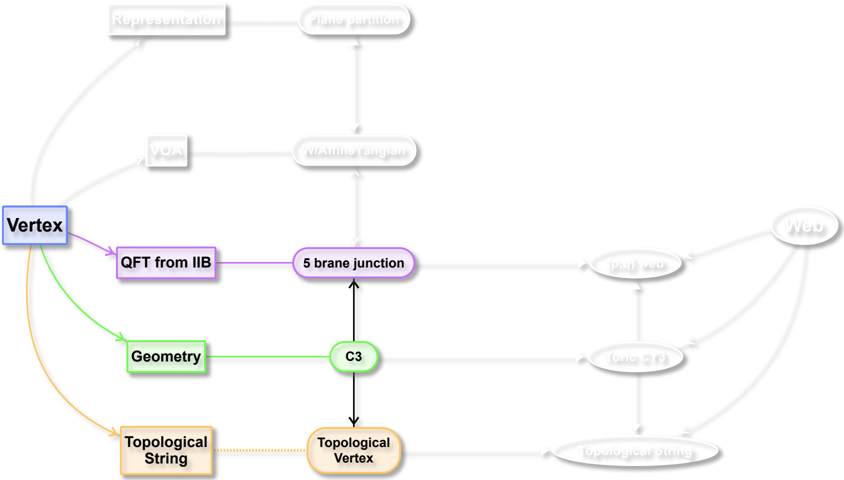
Gaiotto Rapcak '17

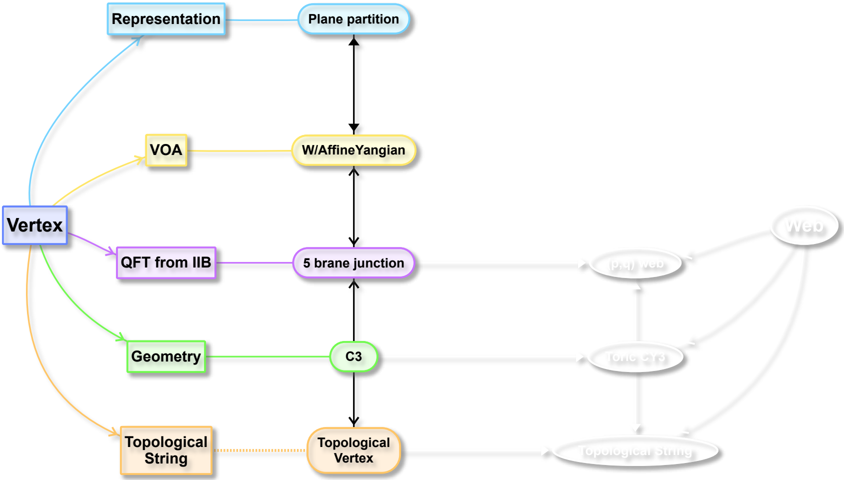


$\times \underset{x_0, x_1}{C} \times \underset{x_7, x_8, x_9}{R^3}$

picture: Gaiotto Rapcak '17

conjecture: VOA on the 2D junction of 4D QFT


Representation
Plane partition
VOA
W/Affine Yangian
Vertex
QFT from IIB
5 brane junction
Web
(p,q) web
Geometry
C3
Toric CY3
Topological String
Topological Vertex
Topological String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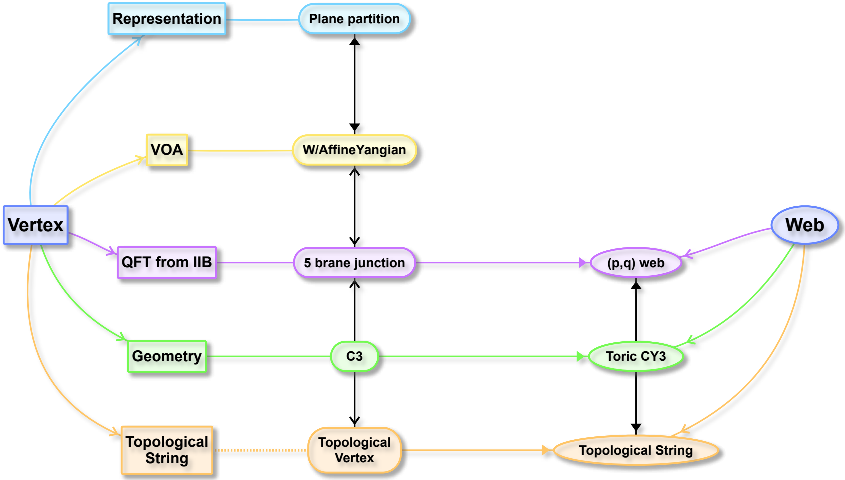
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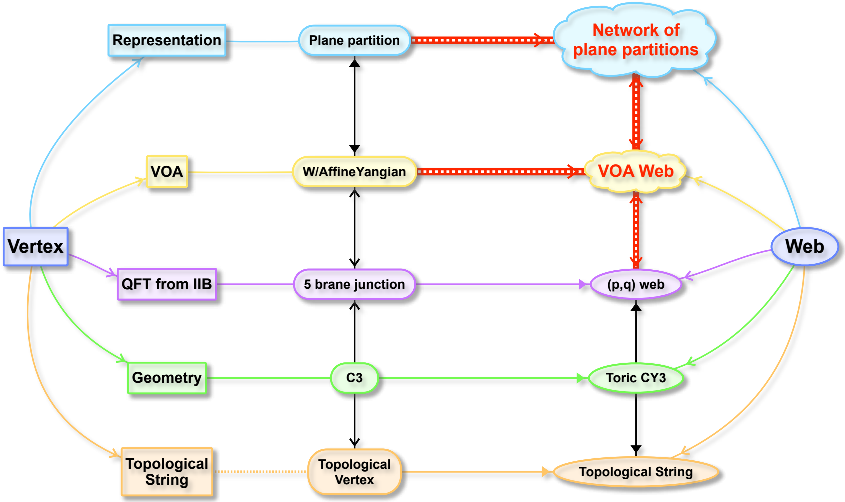
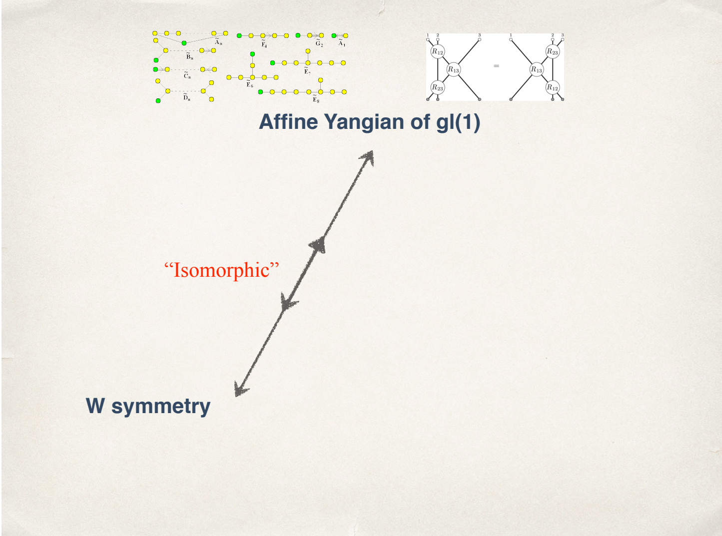

Representation
Plane partition
Network of plane partitions
VOA
W/AffineYangian
VOA Web
Vertex
Web
QFT from IIB
5 brane junction
(p,q) web
Geometry
C3
Toric CY3
Topological String
Topological Vertex
Topological String

Outline

Relation between W algebra and affine Yangian



Modes of $\mathcal{W}_{1+\infty}$

$$W^{(s)}(z) = \sum_{n\in\mathbb{Z}} \frac{W^{(s)}_n}{z^{n+s}} \qquad s = 1,2,3,\ldots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
spin-5	$\ldots$	X_{-4}	X_{-3}	X_{-2}	X_{-1}	X_0	X_1	X_2	X_3	X_4	$\ldots$
spin-4	$\ldots$	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3	U_4	$\ldots$
spin-3	$\ldots$	W_{-4}	W_{-3}	W_{-2}	W_{-1}	W_0	W_1	W_2	W_3	W_4	$\ldots$
spin-2	$\ldots$	L_{-4}	L_{-3}	L_{-2}	L_{-1}	L_0	L_1	L_2	L_3	L_4	$\ldots$
spin-1	$\ldots$	J_{-4}	J_{-3}	J_{-2}	J_{-1}	J_0	J_1	J_2	J_3	J_4	$\ldots$

Regrouping the modes

$$W^{(s)}(z) = \sum_{n\in\mathbb{Z}} \frac{W_n^{(s)}}{z^{n+s}} \qquad s = 1, 2, 3, \ldots$$

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
spin-5	$\ldots$	X_{-3}	X_{-2}	$X_{-1}\sim e_4$	$X_0\sim \psi_5$	$X_1\sim f_4$	X_2	X_3	X_4
spin-4	$\ldots$	U_{-3}	U_{-2}	$U_{-1}\sim e_3$	$U_0\sim \psi_4$	$U_1\sim f_3$	U_2	U_3	U_4
spin-3	$\ldots$	W_{-3}	W_{-2}	$W_{-1}\sim e_2$	$W_0\sim \psi_3$	$W_1\sim f_2$	W_2	W_3	W_4
spin-2	$\ldots$	L_{-3}	L_{-2}	$L_{-1}\sim e_1$	$L_0\sim \psi_2$	$L_1\sim f_1$	L_2	L_3	L_4
spin-1	$\ldots$	J_{-3}	J_{-2}	$J_{-1}\sim e_0$	$J_0\sim \psi_1$	$J_1\sim f_0$	J_2	J_3	J_4

affine Yangian generators

$$e(z) = \sum_{j=0}^{\infty} \frac{e_j}{z^{j+1}} \qquad \psi(z) = 1 + \sigma_3 \sum_{j=0}^{\infty} \frac{\psi_j}{z^{j+1}} \qquad f(z) = \sum_{j=0}^{\infty} \frac{f_j}{z^{j+1}}$$

Affine Yangian of $\mathfrak{gl}_1$

Def: Associative algebra with generators e_j, f_j and ψ_j, $j = 0, 1, \ldots$

- Generators

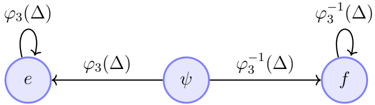
$$\psi(z) = 1 + (h_1 h_2 h_3) \sum_{j=0}^{\infty} \frac{\psi_j}{z^{j+1}} \qquad e(z) = \sum_{j=0}^{\infty} \frac{e_j}{z^{j+1}} \qquad f(z) = \sum_{j=0}^{\infty} \frac{f_j}{z^{j+1}}$$

- Parameters (h_1, h_2, h_3) with $h_1 + h_2 + h_3 = 0$
- One $\mathcal{S}_3$ invariant function $\varphi(z) = \frac{(z+h_1)(z+h_2)(z+h_3)}{(z-h_1)(z-h_2)(z-h_3)}$
- Defining relations

$$[e(z)\,, f(w)] = -\frac{1}{h_1 h_2 h_3} \frac{\psi(z) - \psi(w)}{z - w}$$

$$\psi(z)\, e(w) \sim \varphi(z-w)\, e(w)\, \psi(z) \qquad \psi(z)\, f(w) \sim \varphi(w-z)\, f(w)\, \psi(z)$$

$$e(z)\, e(w) \sim \varphi(z-w)\, e(w)\, e(z) \qquad f(z)\, f(w) \sim \varphi(w-z)\, f(w)\, f(z)$$



Affine Yangian of $\mathfrak{gl}_1$

In terms of modes e_j, f_j and ψ_j, $j = 0, 1, \ldots$

$$
\begin{aligned}
0 &= [\psi_j, \psi_k] \\
\psi_{j+k} &= [e_j, f_k] \\
\sigma_3\{\psi_j, e_k\} &= [\psi_{j+3}, e_k] - 3[\psi_{j+2}, e_{k+1}] + 3[\psi_{j+1}, e_{k+2}] - [\psi_j, e_{k+3}] \\
&\quad + \sigma_2[\psi_{j+1}, e_k] - \sigma_2[\psi_j, e_{k+1}] \\
-\sigma_3\{\psi_j, f_k\} &= [\psi_{j+3}, f_k] - 3[\psi_{j+2}, f_{k+1}] + 3[\psi_{j+1}, f_{k+2}] - [\psi_j, f_{k+3}] \\
&\quad + \sigma_2[\psi_{j+1}, f_k] - \sigma_2[\psi_j, f_{k+1}] \\
\sigma_3\{e_j, e_k\} &= [e_{j+3}, e_k] - 3[e_{j+2}, e_{k+1}] + 3[e_{j+1}, e_{k+2}] - [e_j, e_{k+3}] \\
&\quad + \sigma_2[e_{j+1}, e_k] - \sigma_2[e_j, e_{k+1}] \\
-\sigma_3\{f_j, f_k\} &= [f_{j+3}, f_k] - 3[f_{j+2}, f_{k+1}] + 3[f_{j+1}, f_{k+2}] - [f_j, f_{k+3}] \\
&\quad + \sigma_2[f_{j+1}, f_k] - \sigma_2[f_j, f_{k+1}]
\end{aligned}
$$

with

$$
h_1 + h_2 + h_3 = 0 \qquad \sigma_2 \equiv h_1 h_2 + h_2 h_3 + h_1 h_3 \qquad \sigma_3 \equiv h_1 h_2 h_3
$$

Schiffmann Vasserot '12 *Maulik Okounkov '12*

Feigin Jimbo Miwa Mukhin '10-11

Tsymbaliuk '14

W algebra and affine Yangian

$$\mathcal{Y}[\widehat{\mathfrak{gl}_1}] \cong \mathrm{UEA}[\mathcal{W}_{1+\infty}[\lambda]]$$

Procházka '15

Gaberdiel Gopakumar Li Peng '17

for q-version $\mathcal{U}[\widehat{\widehat{\mathfrak{gl}_1}}] \cong \mathrm{UEA}[\text{q-}\mathcal{W}_{1+\infty}[\lambda]]$

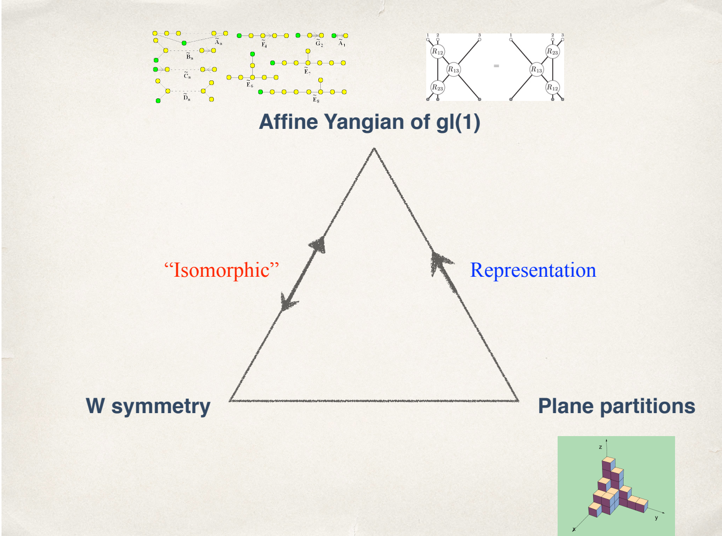
Miki '07

Feigin Jimbo Miwa Mukhin '10-11

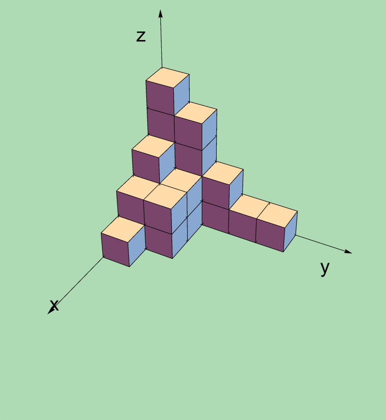
Advantages of affine Yangian over $\mathcal{W}_\infty$

1. number of generators
   - $\mathcal{W}_\infty$: ∞
   $$J(z)\,,T(z)\,,W^{(3)}(z)\,,W^{(4)}(z)\;\ldots$$
   - affine Yangian of $\mathfrak{gl}_1$: only 3
   $$\psi(z)\,,e(z)\,,f(z)$$
2. Defining relations
   - $\mathcal{W}_\infty$:
   non-linear, fixed order by order by Jacobi-identities
   - affine Yangian of $\mathfrak{gl}_1$:
   linear, given explicitly
3. $\mathcal{S}_3$ invariance
   - $\mathcal{W}_\infty$: Hidden
   - affine Yangian of $\mathfrak{gl}_1$: manifest

Plane partition as representations of affine Yangian

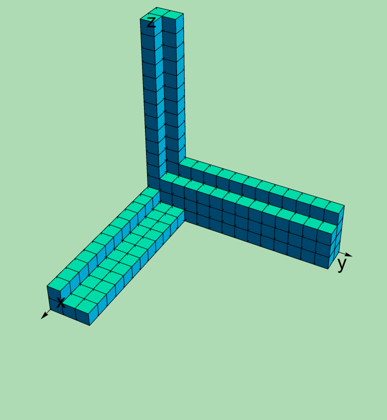


Plane partition via box stacking



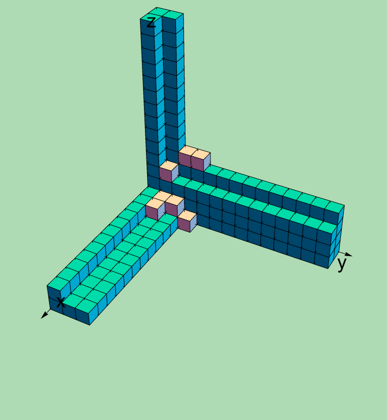
Plane partition with non-trivial asymptotics

Ground state of $(\Lambda_x, \Lambda_y, \Lambda_z)$

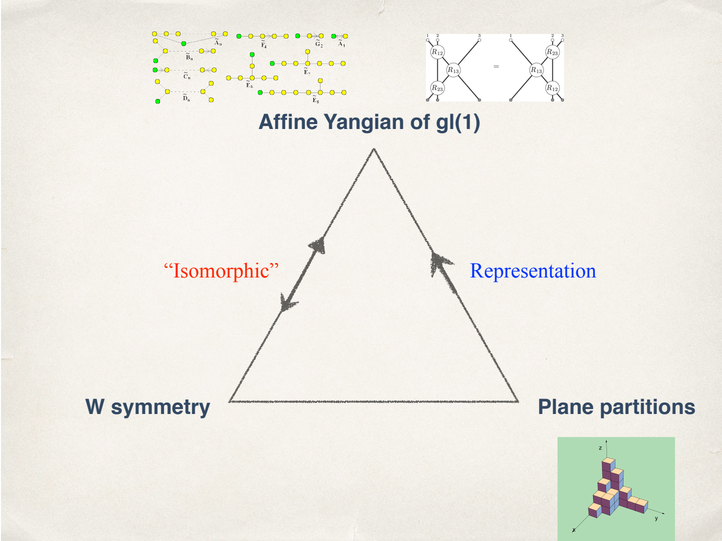


Plane partition with non-trivial asymptotics

a level-7 excited states of $(\Lambda_x, \Lambda_y, \Lambda_z)$



Plane partitions are faithful representations of $\hat{\mathcal{Y}}(\mathfrak{gl}_1)$



Action of $\hat{\mathcal{Y}}(\mathfrak{gl}_1)$ on a plane partition

- $\psi(z)$ acts diagonally *Tsymbaliuk '14, Prochazka '15*

$$\psi(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$$

$$\psi_\Lambda(z) \equiv \left(1+\frac{\psi_0\sigma_3}{z}\right) \prod_{\square\in(\Lambda)} \varphi(z-h(\square))$$

$$h(\square) = h_1 x(\square) + h_2 y(\square) + h_3 z(\square)$$

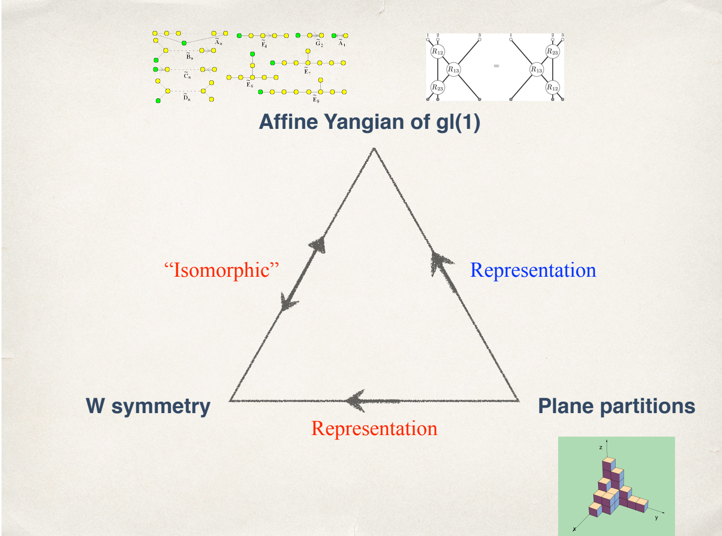
- $e(z)$ adds one box

$$e(z)|\Lambda\rangle = \sum_{\square\in \mathrm{Add}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)}\psi_\Lambda(w)\right]^{\frac{1}{2}}}{z-h(\square)}|\Lambda+\square\rangle$$

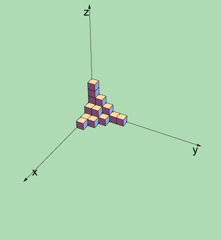
- $f(z)$ removes one box

$$f(z)|\Lambda\rangle = \sum_{\square\in \mathrm{Rem}(\Lambda)} \frac{\left[-\frac{1}{\sigma_3}\mathrm{Res}_{w=h(\square)}\psi_\Lambda(w)\right]^{\frac{1}{2}}}{z-h(\square)}|\Lambda-\square\rangle$$

plane partition as representations

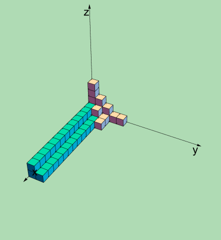


Plane partition as representations of W



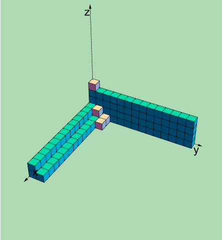
Trivial b.c.

vacuum



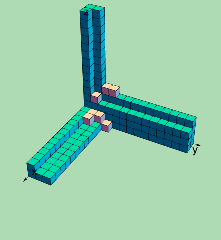
$(\Lambda_x; 0) = (\Lambda; 0)$

perturbative
in Vasiliev



$(\Lambda_x; \Lambda_y) = (\Lambda_+; \Lambda_-)$

non-perturbative
in Vasiliev

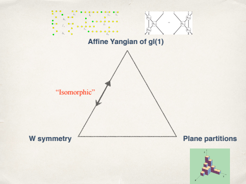


$(\Lambda_x; \Lambda_y; \Lambda_z)$

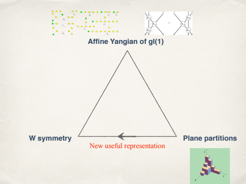
new representation

character of $\mathcal{W}_{1+\infty}$= generating function of plane partition

Application



- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



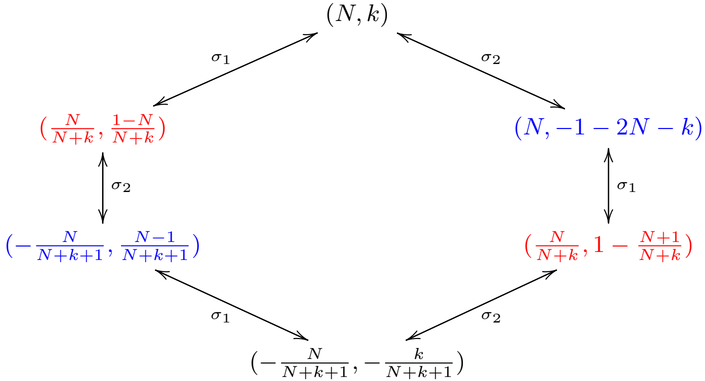
- Character computation more transparent

$\mathcal{S}_3$ action on $\mathcal{W}_{N,k}$ coset

$\mathcal{W}_{N,k}$ coset

$$\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

had hidden $\mathcal{S}_3$

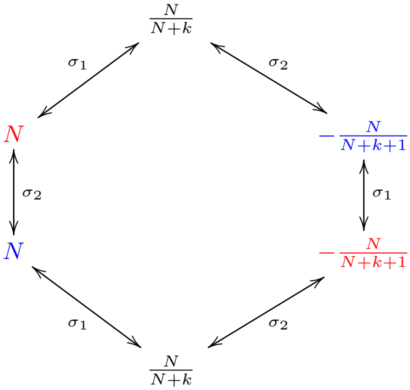


$\mathcal{S}_3$ action on 't Hooft coupling

$\mathcal{W}_{N,k}$ coset

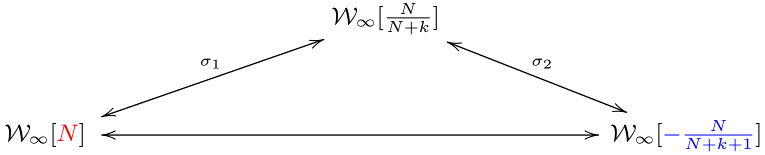
$$\frac{\mathfrak{su}(N)_k \oplus \mathfrak{su}(N)_1}{\mathfrak{su}(N)_{k+1}}$$

't Hooft coupling $\lambda = \frac{N}{N+k}$ transform under $\mathcal{S}_3$



Triality symmetry for higher spin holography

For fixed c, three $\mathcal{W}_\infty[\lambda]$ are isomorphic *Gaberdiel Gopakumar '12*



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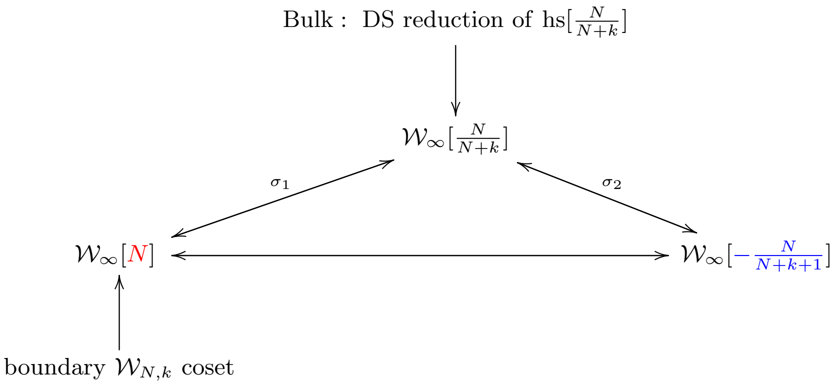


Crucial in Higher spin $\mathrm{AdS}_3/\mathrm{CFT}_2$ (Vasiliev theory in $\mathrm{AdS}_3 = \mathcal{W}_{N,k}$ coset)

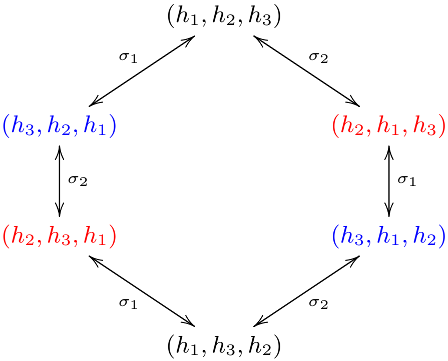
- $\mathcal{S}_3$ symmetry in $\mathcal{W}_\infty$CFT is highly non-trivial
  - hard to check/prove *Gaberdiel Gopakumar '12, Linshaw '17*
  - UV — IR
- Manifest in $\mathcal{Y}[\widehat{\mathfrak{gl}_1}]$

$\mathcal{Y}[\widehat{\mathfrak{gl}_1}]$ depends on (h_1, h_2, h_3) symmetrically

$$h_1 = -\sqrt{\frac{N+k+1}{N+k}} \qquad h_2 = \sqrt{\frac{N+k}{N+k+1}} \qquad h_3 = \frac{1}{\sqrt{(N+k)(N+k+1)}}$$

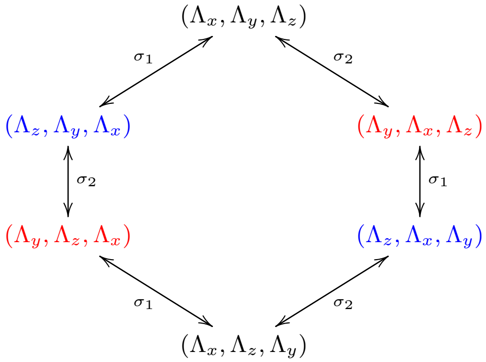
Procházka '15, *Gaberdiel Gopakumar Li Peng '17*

Under $\mathcal{S}_3$ transformation on (N, k)

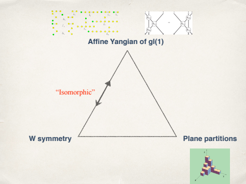


$\mathcal{S}_3$ symmetry of plane partition

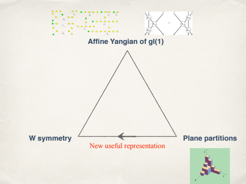
The representations of $\mathcal{W}_\infty$ comes in $\mathcal{S}_3$ family



Application



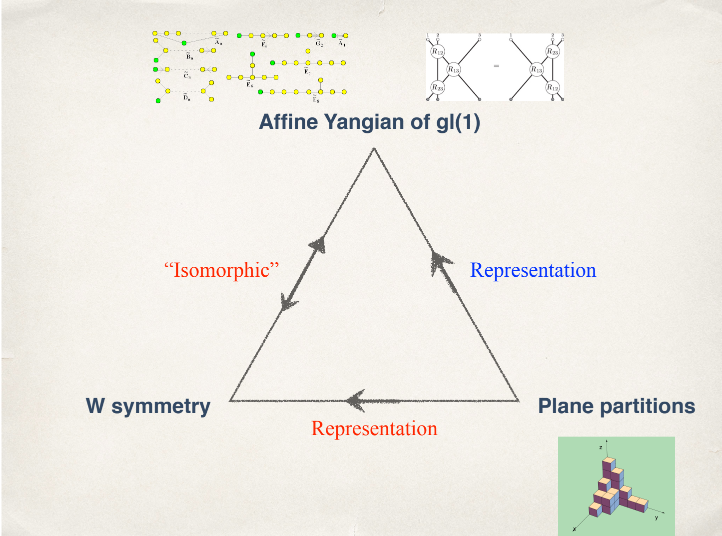
- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



- Character computation more transparent

Outline

Bosonic W and affine Yangian



Two questions

1. Supersymmetrize $\triangle$?

2. $\triangle$ as lego pieces for new VOA/affine Yangian?

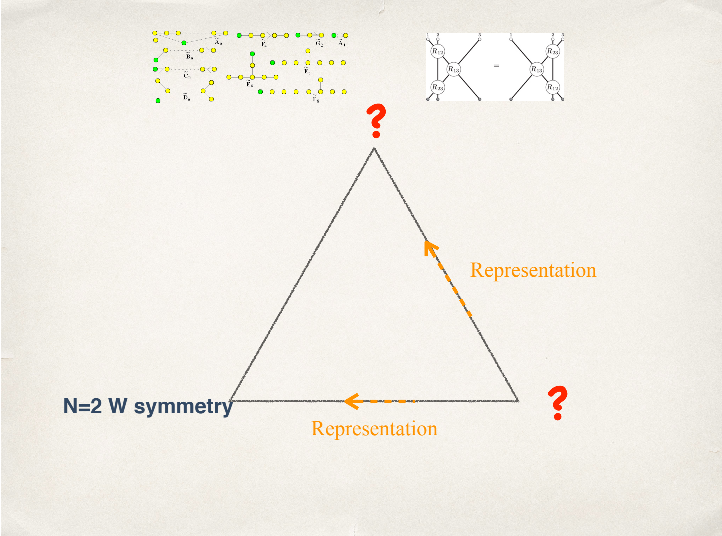
Rapcak Prochazka '17, Gaberdiel Li Peng Zhang'17

A surprising (partial) answer

Glue two $\triangle$ to get $\mathcal{N}=2$ version of $\triangle$

Gaberdiel Li Peng Zhang'17

$\mathcal{N} = 2$ version?



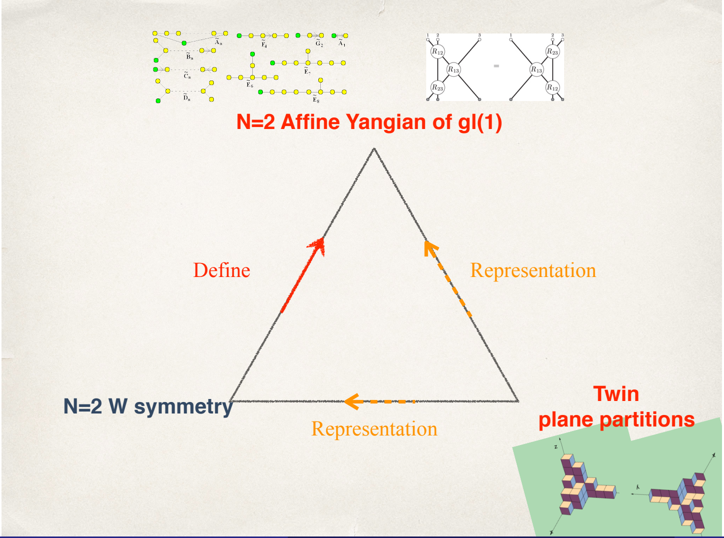
Constructing $\mathcal{N}=2$ version

1. Rewrite representations of $\mathcal{N}=2\ \mathcal{W}_\infty$ in terms of (some version) of plane partitions

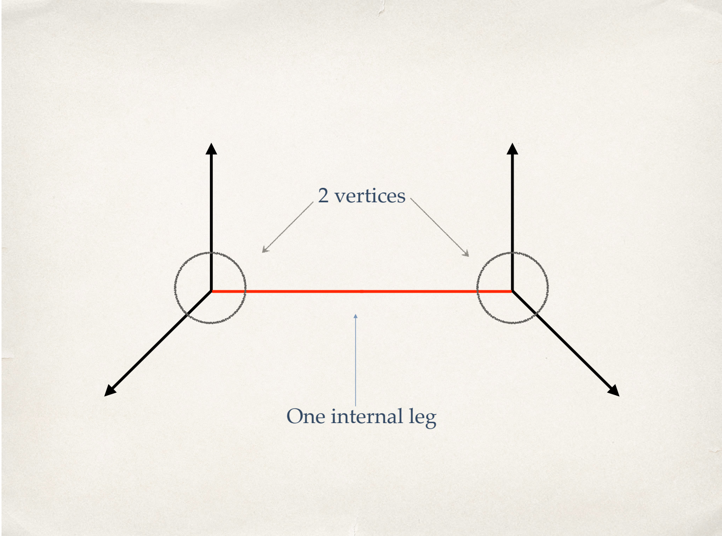
Twin plane partition

2. Define $\mathcal{N}=2$ affine Yangian such that
   - twin plane partitions are faithful representations
   - reproduce $\mathcal{N}=2\ \mathcal{W}_\infty$ charges

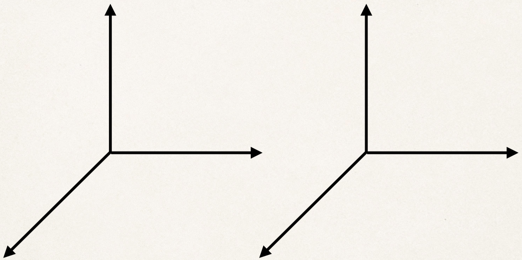
$\mathcal{N}=2$ version



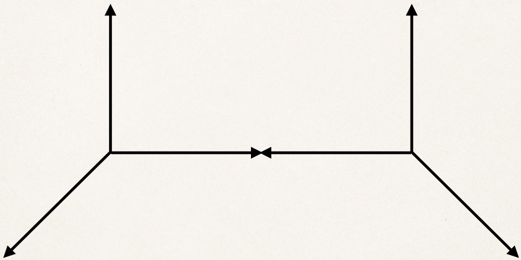
Simplest gluing: 2 vertices and 1 internal leg



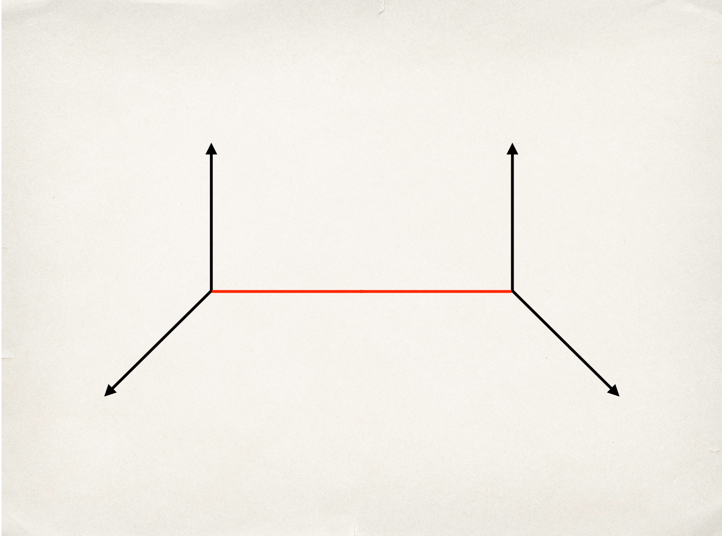
Two copies: left and right



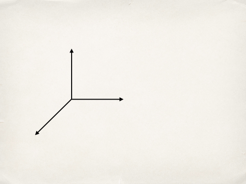
Gluing: two external legs facing opposite directions



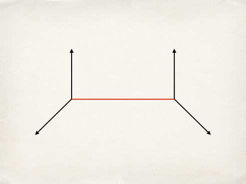
Gluing: two external legs fuse and become internal leg



Building blocks and gluing



1. Algebra: $\mathcal{W}_{1+\infty}$ $\Rightarrow$ affine Yangian of $\mathfrak{gl}_1$
2. Representation: plane partitions



1. Algebra: internal leg $\Rightarrow$ additional operators
2. Representation:

   bi-module: change b.c. for both vertices

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$

Gaberdiel Li Peng Zhang '17

1. Bosonic sub-algebra

$$\mathcal{W}_{1+\infty}[\lambda] \quad \oplus \quad \mathcal{W}_{1+\infty}[1-\lambda]$$

2. Fermions:

$$(\rho \quad , \quad \overline{\rho^t})$$

$$(\overline{\rho^t} \quad , \quad \rho)$$

Decomposing $\mathcal{N}=2$ $\mathcal{W}_\infty[\lambda]$

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1. Bosonic sub-algebra

$$\begin{array}{ccc} \mathcal{W}_{1+\infty}[\lambda] & \oplus & \mathcal{W}_{1+\infty}[1-\lambda] \\ \Downarrow & & \Downarrow \\ \widehat{\mathcal{Y}(\mathfrak{gl}_1)} & \oplus & \widehat{\mathcal{Y}(\mathfrak{gl}_1)} \\ \Downarrow & & \Downarrow \\ \text{Left plane partition} & & \text{Right plane partition} \end{array}$$

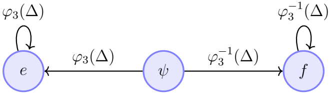
2. Fermions: $(\rho \quad , \quad \overline{\rho^t})$

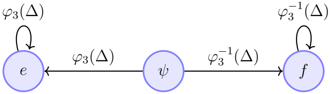
$$(\overline{\rho^t} \quad , \quad \rho)$$

internal legs $\Longrightarrow$ additional operators

TPP building blocks $\Longrightarrow$ yangian generators

Bosonic sub-algebra $\widehat{\mathcal{Y}(\mathfrak{gl}_1)} \oplus \widehat{\mathcal{Y}(\mathfrak{gl}_1)}$





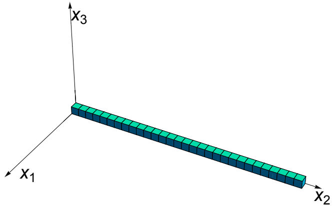
- ψ: Cartan of left $\widehat{\mathcal{Y}(\mathfrak{gl}_1)}$
- e/f: adds/removes $\square$

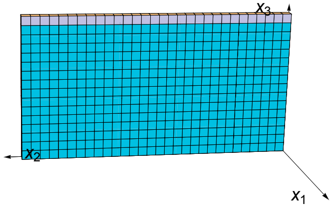
- $\hat{\psi}$: Cartan of right $\widehat{\mathcal{Y}(\mathfrak{gl}_1)}$
- $\hat{e}/\hat{f}$: adds/removes $\widehat{\square}$

Fermions = internal legs = additional operators

- x/y: adds/removes $\blacksquare \equiv (\square, \overline{\square})$
- $\bar{x}/\bar{y}$: adds/removes $\overline{\blacksquare} \equiv (\square, \overline{\square})$

Fermionic building block-1: $\mathbf{x} \equiv \blacksquare \equiv (\square, \bar{\square})$

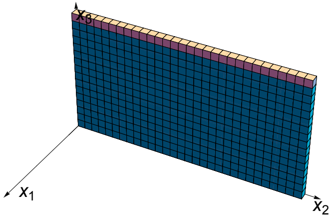


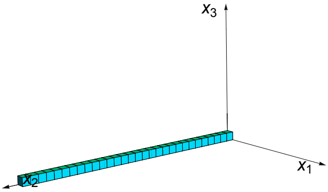


$$h = \frac{1}{2}(1+\lambda) \qquad \hat{h} = \frac{1}{2}\big(1+(1-\lambda)\big)$$

$$\boxed{h + \hat{h} = \frac{3}{2}}$$

Fermionic building block-2: $\bar{\mathrm{x}} \equiv \overline{\blacksquare} \equiv (\bar{\square}, \square)$





$$h = \frac{1}{2}\big(1 + (1-\lambda)\big) \qquad \hat{h} = \frac{1}{2}(1+\lambda)$$

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Building blocks of bosonic affine Yangian of $\mathfrak{gl}_1$

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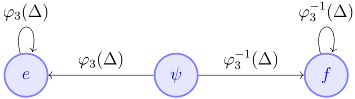
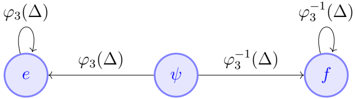
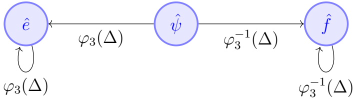
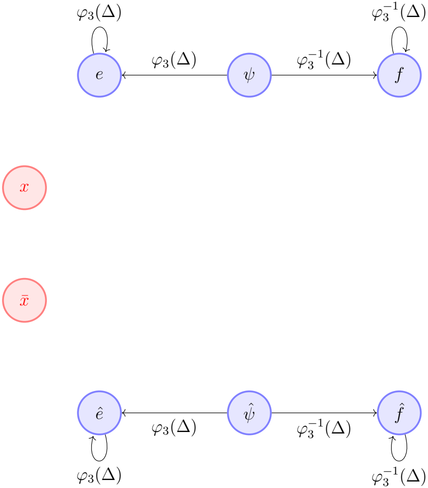


A pair of bosonic affine Yangian of $\mathfrak{gl}_1$





Building blocks of $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$



Constructing $\mathcal{N}=2$ version

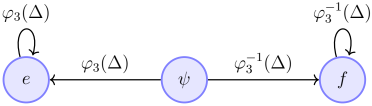
1. Rewrite representations of $\mathcal{N}=2$ $\mathcal{W}_\infty$ in terms of (some version) of plane partitions

   $$\text{Twin plane partition}$$

2. Define $\mathcal{N}=2$ affine Yangian such that

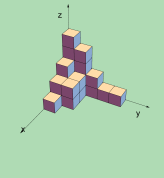
   - twin plane partitions are faithful representations
   - reproduce $\mathcal{N}=2$ $\mathcal{W}_\infty$ charges

Bosonic affine Yangian: $\varphi_3(z)$ plays central role



$$\psi(z)\,e(w) \sim \varphi_3(z-w)\,e(w)\,\psi(z) \qquad \psi(z)\,f(w) \sim \varphi_3(w-z)\,f(w)\,\psi(z)$$
$$e(z)\,e(w) \sim \varphi_3(z-w)\,e(w)\,e(z) \qquad f(z)\,f(w) \sim \varphi_3(w-z)\,f(w)\,f(z)$$

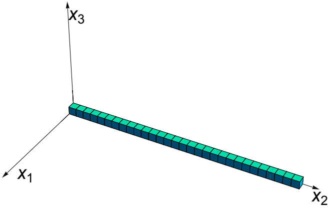
$$\boxed{\varphi_3(z) = \frac{(z+h_1)(z+h_2)(z+h_3)}{(z-h_1)(z-h_2)(z-h_3)}}$$

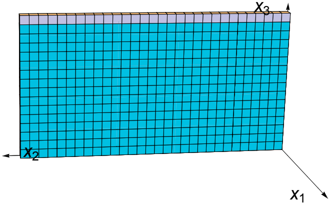


- $\psi(z)|\Lambda\rangle = \psi_\Lambda(z)|\Lambda\rangle$

$$\boxed{\psi_\Lambda(z) \equiv \left(1+\frac{\psi_0\sigma_3}{z}\right)\prod_{\square\in\Lambda}\varphi_3(z-h(\square))}$$

Internal leg: $\varphi_2(z)$ build directly from $\varphi_2(z)$





$$\begin{cases} \psi(z) & = \left(1+\frac{\psi_0\sigma_3}{z}\right)\prod_{n=0}^{\infty}\varphi_3(z-nh_2) = \left(1+\frac{\psi_0\sigma_3}{z}\right)\varphi_2(z) \\ \hat{\psi}(z) & = \left(1+\frac{\psi_0\sigma_3}{z}\right)\varphi_2^{-1}(-z-\sigma_3\hat{\psi}_0) \end{cases}$$

$$\boxed{\varphi_2(z) = \frac{z(z+h_2)}{(z-h_1)(z-h_3)}}$$

Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

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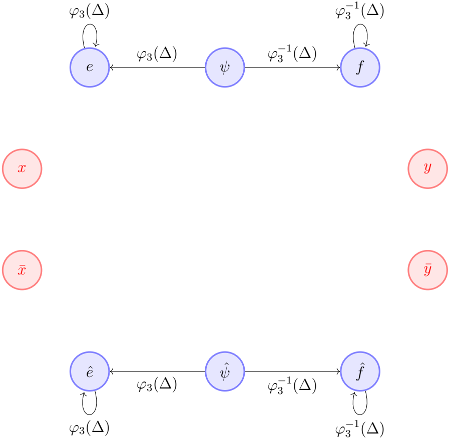

Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

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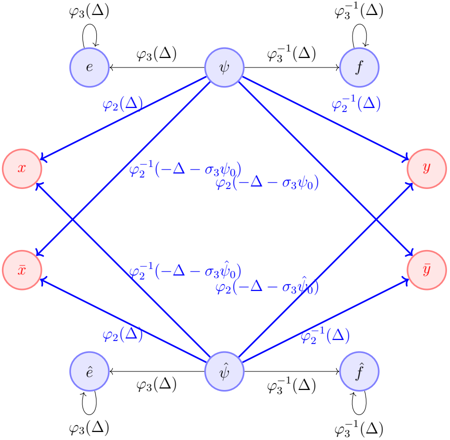

Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

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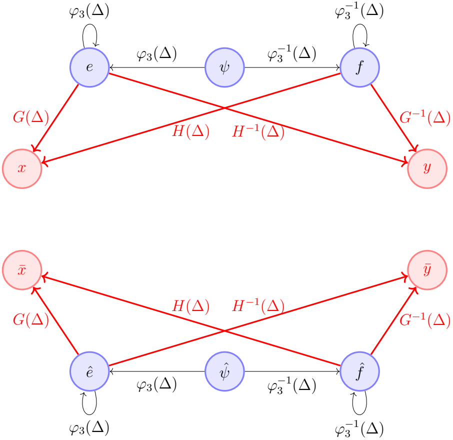

Building $\mathcal{N}=2$ affine Yangian of $\mathfrak{gl}_1$

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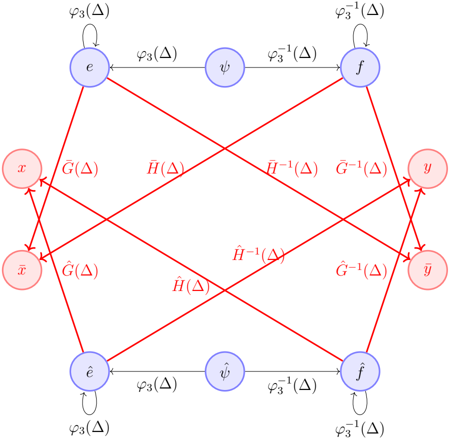
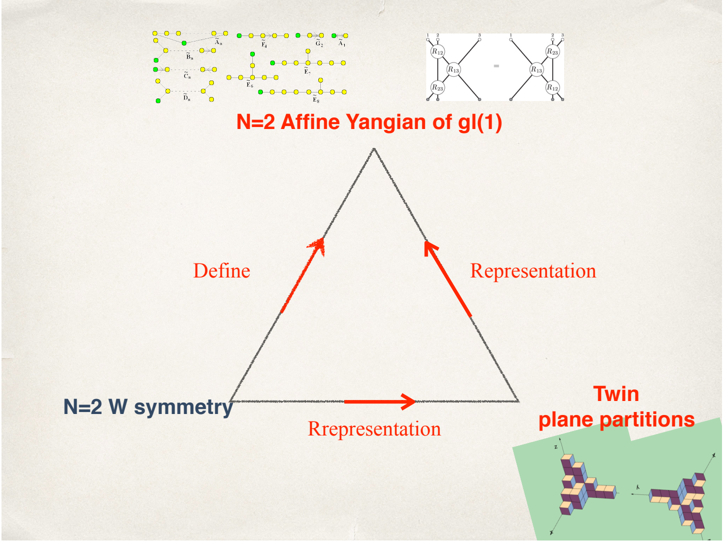


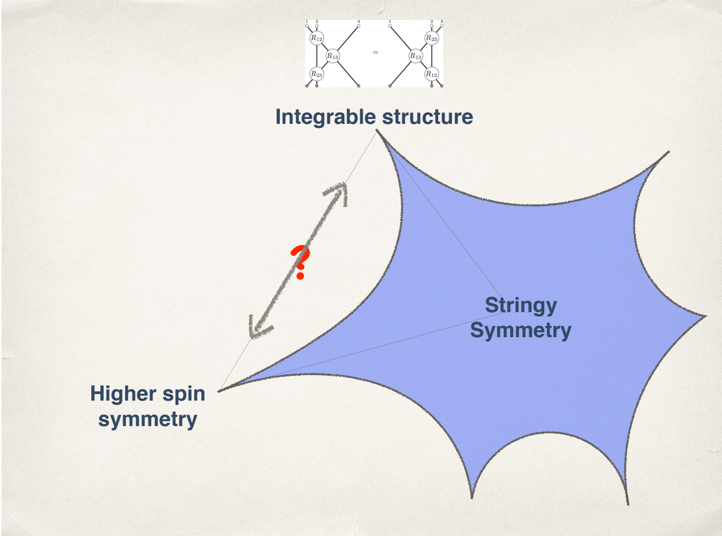

N=2 Affine Yangian of gl(1)
Define
Representation
N=2 W symmetry
Rrepresentation
Twin
plane partitions

Lessons

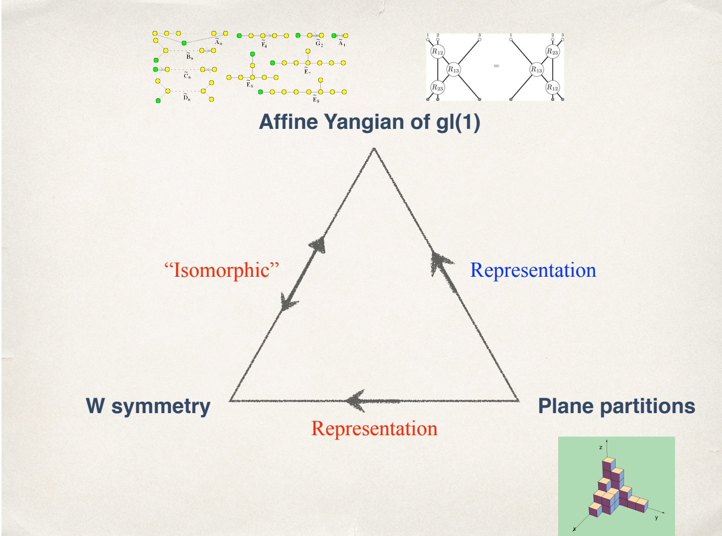
- plane partition is also very useful in the gluing process
  - visualize Fock space
  - Define algebra by faithful representation

Outline

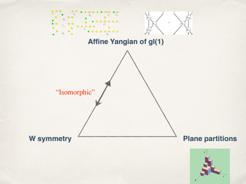
HS and integrability within stringy symmetry



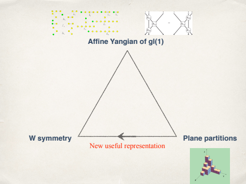
W — affine Yangian — Plane partition



Applications of bosonic triangle

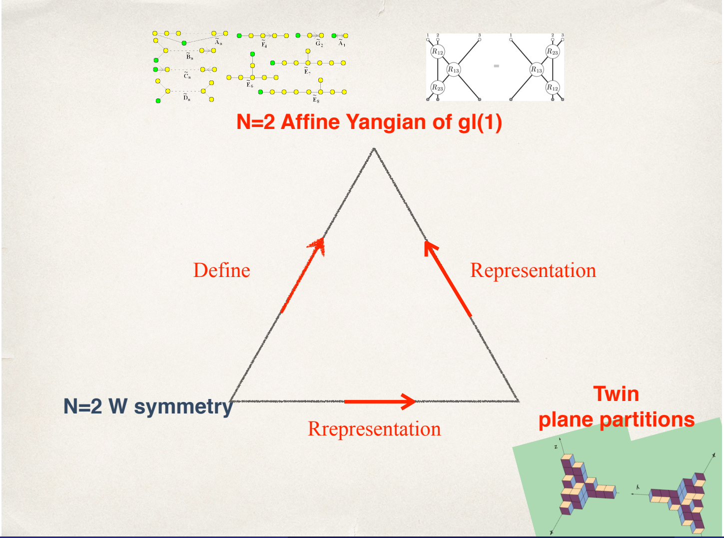


- Make S_3 symmetry in $\mathcal{W}$ CFT manifest



- Character computation more transparent

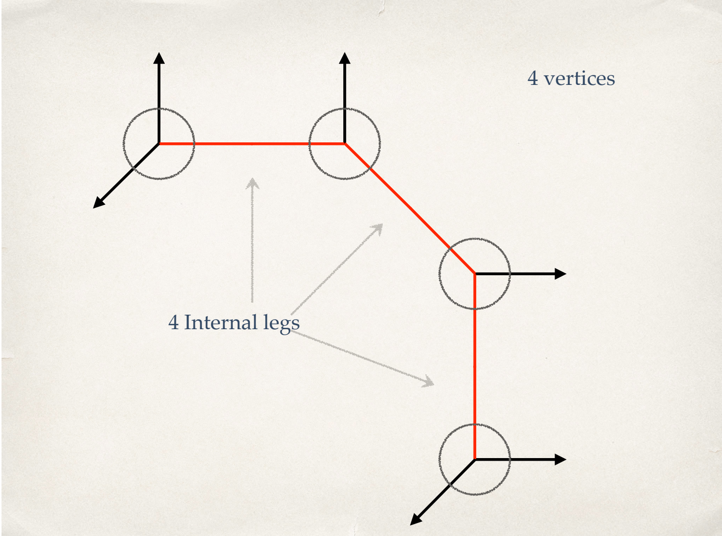
New affine Yangian via gluing



Open problems

1. large $\mathcal{N}=4\ \mathcal{W}_\infty[\lambda]$
2. Classification of affine Yangians from gluing
3. Gluing of finite truncations

Gluing example: 4 vertices and 3 internal legs



More open problems

1. Deeper relation between higher spin symmetry and integrable structure ?
2. Mathematical description of stringy symmetry?
3. Application of stringy symmetry?

Thank you very much !