On the open string pair production enhancement

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1809.03806 (to appear in PLB), Qiang Jia,JXL;
1901.XXXXX, Qiang Jia, JXL, Zhihao Wu& Xiaoyin Zhu

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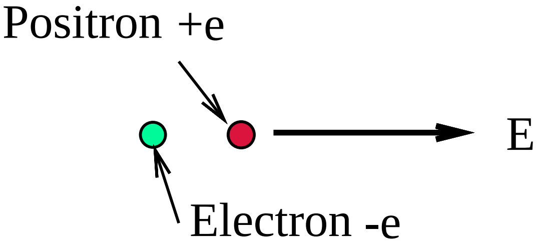
QED Vacuum Fluctuations

Applying a constant E to QED vacuum, there is certain probability to create real electron and positron pairs from the vacuum fluctuations, called Schwinger pair production (1951).



$$(E \sim 10^{18}\, V/m)$$

The current lab E-field limit: $\sim 10^{10}\, V/m$

Stringy process

The natural question is

- Does there exist an analogous process in string theory?

Stringy process

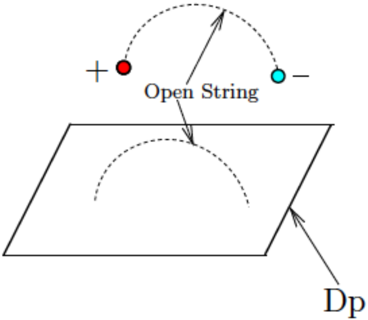
For unoriented string (bosonic and Type I superstring), this was considered by Bachas-Porrati'92.

We will focus in this talk on the open string pair production for systems of D-branes,

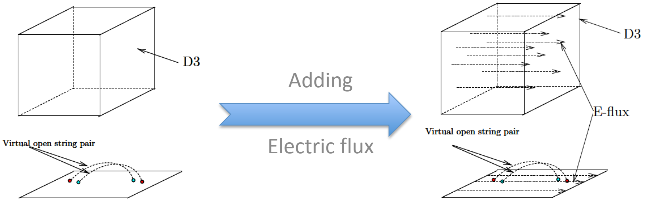
carrying constant electric and magnetic fluxes,

in oriented Type II string theories and the resulting pair production can be significant under certain conditions.

D-branes



Take our 4-dim world as a D3 carrying an electric field



A single D3 brane carrying an electric field

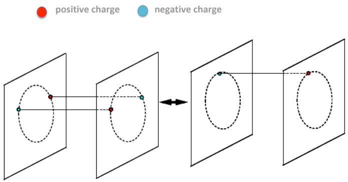
In a sharp contrast to the Schwinger pair production in QED, there is no pair production here.

This is due to each virtual open string being charge neutral (zero net charge) and their two ends experiencing the same electric field.

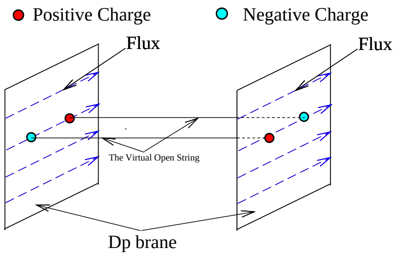
The open string pair production

In order to have the pair production, a possible choice is to let the two ends of the virtual charge-neutral open string experience different electric field.

A simple setup for this is to consider two Dp branes in Type II string theory, placed parallel at a separation, with each carrying a different electric field.



The open string pair production



Stringy computations show indeed a non-vanishing pair production rate for this setup. However, this rate is usually vanishing small for any realistic electric fields and so has no any practical use.

This rate can be greatly enhanced if we add in addition a magnetic flux in a particular manner on each Dp.

The pair production rate

For this purpose, consider the electric/magnetic tensor $\hat{F}^1$ on one Dp brane and the $\hat{F}^2$ on the other Dp brane, respectively, as

$$\hat{F}^a = \begin{pmatrix} 0 & -f_a & 0 & 0 & 0 & \dots \\ f_a & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & -g_a & 0 & \dots \\ 0 & 0 & g_a & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}_{(p+1)\times(p+1)}, \qquad (2.1)$$

where f_a denotes the electric field ($|f_a| < 1$) while g_a the magnetic one ($|g_a| < \infty$) with $a = 1, 2$, and $6 \geq p \geq 3$. Note $\hat{F} = 2\pi\alpha' F$.

The pair production rate

For having the open string pair production rate, we first need to compute the open string one-loop annulus amplitude between the two Dp Lu'17. It is

$$\Gamma = \frac{2^2\,|f_1-f_2||g_1-g_2|}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty \frac{dt}{t^{\frac{p-1}{2}}}e^{-\frac{y^2 t}{2\pi\alpha'}}\frac{(\cosh\pi\nu_0' t-\cos\pi\nu_0 t)^2}{\sin\pi\nu_0 t\sinh\pi\nu_0' t}$$
$$\times\prod_{n=1}^{\infty}\frac{[(1-2|z|^{2n}e^{-\pi\nu_0' t}\cos\pi\nu_0 t+|z|^{4n}e^{-2\pi\nu_0' t})(1-2|z|^{2n}e^{\pi\nu_0' t}\cos\pi\nu_0 t+|z|^{4n}e^{2\nu_0' t})]^2}{(1-|z|^{2n})^6(1-2|z|^{2n}\cos 2\pi\nu_0 t+|z|^{4n})(1-2|z|^{2n}\cosh 2\pi\nu_0' t+|z|^{4n})} \quad (2.2)$$

where y the brane separation, $|z| = e^{-\pi t} < 1$, and the electric parameter $\nu_0 \in [0,\infty)$ and the magnetic one $\nu_0' \in [0,1/2]$ are given by respective fluxes as

$$\tanh\pi\nu_0 = \frac{|f_1-f_2|}{1-f_1 f_2}, \quad \tan\pi\nu_0' = \left|\frac{g_1-g_2}{1+g_1 g_2}\right|. \quad (2.3)$$

The pair production rate

For large t, the integrand behaves like

$$\sim e^{-\frac{y^2 t}{2\pi\alpha'}} e^{\pi\nu_0' t} = e^{-2\pi t\left[\frac{y^2}{(2\pi)^2\alpha'} - \frac{\nu_0'}{2}\right]}, \qquad (2.4)$$

which blows up when $y \leq \pi\sqrt{2\nu_0'\alpha'}$, indicating a tachyonic instability.

The open string pair production rate

- The integrand of the above amplitude has an infinite number of simple poles occurring on the positive real t-axis at $t_k = k/\nu_0$ with $k = 1, 2, \cdots$.
- Therefore this amplitude has an imaginary part, indicating the decay of the underlying system.
- The dynamical process responsible for this decay is the open string pair production. It occurs at each of these simple poles.
- The non-perturbative pair production rate is given as sum of the residues of the integrand at thess simple poles times π per unit worldvolume following Bachas-Porrati'92 as

The open string pair production rate

$$
\begin{aligned}
\mathcal{W} &= -2\,\mathrm{Im}\Gamma \\
&= \frac{8\,|f_1 - f_2||g_1 - g_2|}{(8\pi^2\alpha')^{\frac{p+1}{2}}} \sum_{k=1}^{\infty} (-)^{k-1} k \left(\frac{\nu_0}{k}\right)^{\frac{p-3}{2}} e^{-\frac{k y^2}{2\pi\nu_0\alpha'}} \\
&\times \frac{\left[\cosh\frac{k\pi\nu_0'}{\nu_0} - (-)^k\right]^2}{\sinh\frac{k\pi\nu_0'}{\nu_0}} Z_k(\nu_0, \nu_0'), \qquad (2.5)
\end{aligned}
$$

where

$$
Z_k(\nu_0, \nu_0') = \prod_{n=1}^{\infty} \frac{\left[1 - 2(-)^k e^{-\frac{2nk\pi}{\nu_0}} \cosh\frac{k\pi\nu_0'}{\nu_0} + e^{-\frac{4nk\pi}{\nu_0}}\right]^4}{\left[1 - e^{-\frac{2nk\pi}{\nu_0}}\right]^6 \left[1 - e^{-\frac{2k\pi}{\nu_0}(n-\nu_0')}\right] \left[1 - e^{-\frac{2k\pi}{\nu_0}(n+\nu_0')}\right]}. \qquad (2.6)
$$

The pair production rate enhancement

- The rate is highly suppressed by the brane separation y and the integer k.
- For each given f_a and g_a, we can qualitatively understand this by noting that the mass for each produced open string is $kT_f y$ with $T_f = 1/(2\pi\alpha')$. So the larger k or y or both, the larger the mass is and therefore the more difficult the open string can be produced.
- The pure electric case can be obtained by setting $g_1 = g_2 = 0$ for which $\nu_0' = 0$ from (2.3) and the rate is

$$\mathcal{W}_{g_1=g_2=0} = \frac{32\nu_0|f_1 - f_2|}{(8\pi^2\alpha')^{\frac{p+1}{2}}} \sum_{l=1}^{\infty} \frac{1}{(2l-1)^2} \left(\frac{\nu_0}{2l-1}\right)^{\frac{p-3}{2}} e^{-\frac{(2l-1)y^2}{2\pi\alpha'\nu_0}} \times \prod_{n=1}^{\infty} \left(\frac{1 + e^{-\frac{2n(2l-1)\pi}{\nu_0}}}{1 - e^{-\frac{2n(2l-1)\pi}{\nu_0}}}\right)^8 . \tag{2.7}$$

The pair production rate enhancement

- Now if we further set $f_1 = f_2$, i.e., the two ends of the open string experiences the same electric field, then $\nu_0 = 0$ from (2.3) and we have the rate $\mathcal{W}_{g_1=g_2=0} = 0$.
- This is consistent with that no Schwinger-type of pair production of a single D3 brane carrying a constant electric flux mentioned earlier.
- For $f_1 \neq f_2$, the larger f_1 or f_2 is, the larger ν_0 and $|f_1 - f_2|$ are and the larger the rate $\mathcal{W}$ is.
- In general, the presence of magnetic fluxes enhances this rate. We here consider two special cases to show this explicitly.
- The first is the case of $g_1 = g_2 = g \neq 0$, we have from (2.5) and (2.3)

$$\frac{\mathcal{W}_{g_1=g_2=g}}{\mathcal{W}_{g_1=g_2=0}} = 1 + g^2. \tag{2.8}$$

The pair production rate enhancement

- The second is the case of $\nu_0'/\nu_0 \gg 1$. This says $\nu_0 \ll 1$ since $\nu_0' \in [0, 1/2]$, implying $|f_1 - f_2| \ll 1$ from (2.3). For fixed $\nu_0' \in [0, 1/2]$ and a very small ν_0, the rate (2.5) can be approximated by its leading $k = 1$ term as

$$\mathcal{W} \approx \frac{4|f_1 - f_2||g_1 - g_2|}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \, \nu_0^{\frac{p-3}{2}} \, e^{-\frac{y^2}{2\pi\alpha'\nu_0}} \, e^{\frac{\pi\nu_0'}{\nu_0}} . \qquad (2.9)$$

- The zero-magnetic flux rate (2.7) for the same small ν_0 is

$$\mathcal{W}_{g_1=g_2=0} \approx \frac{32\nu_0|f_1 - f_2|}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \, \nu_0^{\frac{p-3}{2}} \, e^{-\frac{y^2}{2\pi\alpha'\nu_0}} . \qquad (2.10)$$

The pair production rate enhancement

- The enhancement is then

$$\frac{\mathcal{W}}{\mathcal{W}_{g_1=g_2=0}} = \frac{|g_1 - g_2|}{8\nu_0}\, e^{\frac{\pi\nu_0'}{\nu_0}} \tag{2.11}$$

which can be huge given $\nu_0'/\nu_0 \gg 1$ and $\nu_0 \ll 1$.

The rate enhancement

Consider now $\nu_0'/\nu_0 \gg 1$ for $p = 3$ as an illustration!

To have a sense of enhancement, let us make a numerical estimation for illustration!

Take $\nu_0 = 0.02$ and $\nu_0' = 0.5$. This can be achieved with a moderate choice of $g_1 = -g_2 = 1$ (noting $|g_a| < \infty$) and $f_1 = 0.2$ with $f_2 = f_1 - \epsilon$ and $|f_1 - f_2| = |\epsilon| \approx \pi\nu_0(1 - f_1^2) = 0.06 \ll 1$.

The enhancement is then

$$\frac{|g_1 - g_2| e^{\frac{\pi\nu_0'}{\nu_0}}}{8\nu_0} = 1.6 \times 10^{35}\text{!!!} \tag{2.12}$$

A huge number!

The rate enhancement

This does not necessarily mean a large pair production rate!

Let us explore the posibility for a large rate (note $p \geq 3$ for the enhancement) .

According to Nikishov'70, the general rate (2.5) should be more properly interpreted as the decay one for the underlying system while the pair production rate is the leading $k = 1$ term,

$$\mathcal{W}^{(1)} = \frac{8\,|f_1 - f_2||g_1 - g_2|}{(8\pi^2\alpha')^{\frac{p+1}{2}}} \nu_0{}^{\frac{p-3}{2}} e^{-\frac{y^2}{2\pi\nu_0\alpha'}} \frac{\left[\cosh\frac{\pi\nu_0'}{\nu_0} + 1\right]^2}{\sinh\frac{\pi\nu_0'}{\nu_0}} \times Z_1(\nu_0, \nu_0'), \quad (2.13)$$

where $Z_1(\nu_0, \nu_0') \approx 1$ for $\nu_0 \ll 1$.

The rate enhancement

It is clear the $p = 3$ gives the largest rate (Lu'18) and the dimensionless rate, say, for $p = 4$ is smaller by a factor of $(\nu_0/4\pi)^{1/2}$ and so on.

Adding more magnetic flux doesn't help (Jia & Lu'18).

Let us give an estimate of this factor and see how large it is.

In practice, we can only control the brane on which we live, not the other brane. So we can set $f_2 = g_2 = 0$, for example, on the other brane.

The rate enhancement

Note $M_s = 1/\sqrt{\alpha'} \sim$ a few TeV upto $10^{16} \sim 10^{17}$ GeV (Berenstein'14),

Schwinger pair production $eE \sim m_e^2 = 2.5 \times 10^{-7}\mathrm{GeV}^2$,

$f_1 = 2\pi\alpha' eE = 2\pi\, m_e^2/M_s^2 \leq\sim 10^{-13} \ll 1$

$$\nu_0 = \frac{|f_1|}{\pi} = 2\frac{m_e^2}{M_s^2} \leq\sim 10^{-13} \to \left(\frac{\nu_0}{4\pi}\right)^{1/2} \sim 10^{-7} \ll 1 \qquad (2.14)$$

In other words, the dimensionless rate for any other $p > 3$ brane is at least smaller than that of the D3-brane by a factor of 10^{-7}!

The enhanced rate

In terms of the lab. field E and B via

$$f_1 = 2\pi\alpha' eE, \qquad g_1 = 2\pi\alpha' eB, \tag{2.15}$$

the pair production rate(2.13) for D3 brane is now $(Z_k(\nu_0, \nu_0') \approx 1)$

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh \frac{\pi B}{E} + 1\right]^2}{\sinh \frac{\pi B}{E}} e^{-\frac{\pi m^2}{eE}}, \tag{2.16}$$

where we have introduced a mass scale $m = T_f y = y/(2\pi\alpha')$.

Keep in mind, we need to have a nearby D3 brane for this rate!

The enhanced rate

Let us try to understand (2.16) a bit more.

In the absence of both E and B, the mass spectrum for the open string connecting the two D3 is

$$\alpha' M^2 = -\alpha' p^2 = \begin{cases} \frac{y^2}{4\pi^2\alpha'} + N_{\rm R} & (\mathrm{R}-\mathrm{sector}), \\ \frac{y^2}{4\pi^2\alpha'} + N_{\rm NS} - \frac{1}{2} & (\mathrm{NS}-\mathrm{sector}), \end{cases} \tag{2.17}$$

where $p = (k, 0)$ with k the momentum along the brane worldvolume directions, $N_{\rm R}$ and $N_{\rm NS}$ are the standard number operators in the R-sector and NS-sector, respectively, as

$$\begin{aligned} N_{\rm R} &= \sum_{n=1}^{\infty}(\alpha_{-n}\cdot\alpha_n + n d_{-n}\cdot d_n), \\ N_{\rm NS} &= \sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n + \sum_{r=1/2}^{\infty} r d_{-r}\cdot d_r. \end{aligned} \tag{2.18}$$

The enhanced rate

The usual massless $8_{\rm F} + 8_{\rm B}$ degrees of freedom (The 4D N $=$ 4 SYM or the 10D N $=$ 1 SYM) become now massive ones, all with mass $T_f y = y/(2\pi\alpha')$ when $y \neq 0$. This just reflects the spontaneously symmetry breaking $U(2) \to U(1) \times U(1)$ when $y = 0 \to y \neq 0$.

The pair production rate (2.16) is obtained in the weak field limit $f_1 \ll 1, g_1 \ll 1$ and all massive other than the lowest $8_{\rm F} + 8_{\rm B} = 16$ dof are dropped since $Z_1 \approx 1$. In other words, only these 16 charged dof actually contributes to this rate.

The enhanced rate

Now consider the presence of magnetic flux B ($\tan \pi\nu_0' = 2\pi\alpha' eB$), the spectrum in the NS-sector is now,

$$\alpha' E_{\rm NS}^2 = (2N+1)\frac{\nu_0'}{2} - \nu_0' S + \alpha' M_{\rm NS}^2, \qquad (2.19)$$

where $b_0^+ b_0 = N$ defines the Landau level, the mass $M_{\rm NS}$, the spin operator S in the 34-direction and the number operator $N_{\rm NS}$ are

$$\begin{aligned}
\alpha' M_{\rm NS}^2 &= \frac{y^2}{4\pi^2\alpha'} + N_{\rm NS} - \frac{1}{2}, \\
S &= \sum_{n=1}^{\infty}(a_n^+ a_n - b_n^+ b_n) + \sum_{r=1/2}^{\infty}(d_r^+ d_r - \tilde{d}_r^+ \tilde{d}_r), \\
N_{\rm NS} &= \sum_{n=1}^{\infty} n(a_n^+ a_n + b_n^+ b_n) + \sum_{r=1/2}^{\infty} r(d_r^+ d_r + \tilde{d}_r^+ \tilde{d}_r) + N_{\rm NS}^{\perp}.
\end{aligned} \qquad (2.20)$$

The enhanced rate

The so-called first Regge trajectory is given by

$$(a_1^+)^{\tilde{n}} d_{1/2}^+ |0\rangle_{\rm NS}, \tag{2.21}$$

For this, we have

$$\alpha' E_{\rm NS}^2 = -\frac{\nu_0'}{2} + (1-\nu_0')(S-1) + \frac{y^2}{4\pi^2\alpha'}, \tag{2.22}$$

where $S = \tilde{n} + 1 \geq 1.$ So the lowest mass is still given by $d_{1/2}^+|0\rangle_{\rm NS}$ with now the spin $S = 1$ and the effective mass $\alpha' E_{\rm NS}^2 = -\nu_0'/2 + y^2/(4\pi^2\alpha')$ with a tachyonic shift $-\nu_0'/2.$

This shift can also be seen from the integrand of the annulus amplitude for large t as $\exp(-2\pi t[-\frac{\nu_0'}{2} + \frac{y^2}{4\pi^2\alpha'}])$ discussed earlier.

The enhanced rate

We now consider weak magnetic field limit of (2.19), i.e., $g_1 = 2\pi\alpha' eB \ll 1$, giving $\nu_0' = 2\alpha' eB$,

$$E_{\rm NS}^2 = (2N+1)eB - 2eBS + M_{\rm NS}^2, \tag{2.23}$$

Comparing this with electrically charged particle with mass m_S and spin S,

$$E_N^2 = (2N+1)eB - g_S eBS + m_S^2, \tag{2.24}$$

with g_S called the gyromagnetic ratio of the particle. If $g_S = 1/S$, called "minimal coupling". For example for spin-1/2 charged particle, $E_N^2 \geq 0$, no possibility for tachyonic instability.

If $g_S \neq 1/S$, called "non-minimal coupling". For example, for non-abelian massive spin-1 gauge field such as the W-boson, $g_S = 2$. Now $E_0^2 = -eB + M_W^2$. If $B \geq M_W^2/e$, then $E_0^2 < 0$, having a tachyonic instability, called Nielsen-Olsen instability.

The enhanced rate

We now compare the open string pair production rate (2.16) with QED charged scalar, spinor and W-boson pair production rate with the same E and B. The present rate is

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh \frac{\pi B}{E} + 1\right]^2}{\sinh \frac{\pi B}{E}} e^{-\frac{\pi m^2}{eE}}, \qquad (2.25)$$

while for the QED scalar Nikishov'70

$$\mathcal{W}_{\mathrm{QEDscalar}} = \frac{(qE)(qB)}{2(2\pi)^2} \operatorname{csch}\left(\frac{\pi B}{E}\right) e^{-\frac{\pi m^2}{eE}}, \qquad (2.26)$$

for spinor

$$\mathcal{W}_{\mathrm{QEDspinor}} = \frac{(eE)(eB)}{(2\pi)^2} \coth\left(\frac{\pi B}{E}\right) e^{-\frac{\pi m_e^2}{eE}}, \qquad (2.27)$$

and for W-boson Kruglov'01,

$$\mathcal{W}_{\mathrm{W-boson}} = \frac{(qE)(qB)}{2(2\pi)^2} \frac{2\cosh \frac{2\pi B}{E} + 1}{\sinh \frac{\pi B}{E}} e^{-\frac{\pi m_W^2}{eE}}. \qquad (2.28)$$

The enhanced rate

Identifying the mass, when $B = 0$, we have

$$\begin{aligned} \mathcal{W}^{(1)} &= 16\,\mathcal{W}_{QEDscalar} = 8\,\mathcal{W}_{QEDspinor} \\ &= \frac{16}{3}\,\mathcal{W}_{\mathrm{W-boson}} = \frac{8(eE)^2}{(2\pi)^2}\,e^{-\frac{\pi m^2}{eE}}, \end{aligned} \tag{2.29}$$

When $B/E \to \infty$ (or large), we have

$$\begin{aligned} \frac{\mathcal{W}^{(1)}}{\mathcal{W}_{QEDscalar}} &= 4\left[\cosh\frac{\pi B}{E} + 1\right]^2 \to e^{2\pi B/E}, \\ \frac{\mathcal{W}^{(1)}}{\mathcal{W}_{QEDspinor}} &= \frac{2\left[\cosh\frac{\pi B}{E} + 1\right]^2}{\cosh\frac{\pi B}{E}} \to e^{\pi B/E}, \\ \frac{\mathcal{W}^{(1)}}{\mathcal{W}_{W-boson}} &= \frac{4\left[\cosh\frac{\pi B}{E} + 1\right]^2}{2\cosh\frac{2\pi B}{E} + 1} \to 1 \end{aligned} \tag{2.30}$$

The enhanced rate

So there is a sharp difference between the present rate and the QED scalar or QED spinor or W-boson one about their dependence on the applied electric and magnetic fields.

Given what has been said, this sharp difference can actually be understood easily.

For weak fields (or the Lab fields), only the lowest-mass charged modes of the open string contribute to this rate, all with the same mass $m = y/(2\pi\alpha')$, which are $8_{\rm F} + 8_{\rm B}$, giving the 4D N $= 4$ massive SYM (5 massive scalars, 4 massive fermions and 1 massive vector).

All the 16 modes having the same mass is due to the underlying unbroken SUSY before and after the spontaneously gauge symmetry breaking $U(2) \to U(1) \times U(1)$, corresponding to $y = 0 \to$y $\neq 0$. The SUSY breaking is due to the added fluxes.

The enhanced rate

Given the above, we expect to have,

$$\mathcal{W}^{(1)} = 5W_{\rm QED-scalar} + 4W_{\rm QED-spinor} + W_{\rm w-boson}, \qquad (2.31)$$

if we take all the modes having the same mass and with the same $E\&B$.

One can check this holds indeed true and this relation explains all the previous results for $B = 0$ and large B/E, respectively.

It is also very satisfied to have this since they are computed complete differently.

Conclusion& Discussion

- The pair production enhancement found is due to the interplay of non-perturbative Schwinger-type pair production and the presence of magnetic flux.
- The enhanced rate is the largest for $p = 3$ and for any $p > 3$, the corresponding rate can be at least seven orders of magnitude smaller. Adding more magnetic fluxes decreases the rate.
- Any possibility of detection for a brane observer? It all depends on the mass scale $m = y/(2\pi\alpha')$. One naturally expects it to be the one related to SUSY breaking. So unless there exists very low energy SUSY, we expect $m >$TeV. Then it is hardly possible for such a detection since this would require either $eE \sim m^2$ or $eB \sim m^2$, which are too large for current lab. limit.

Conclusion & Discussion

- In order to have such a possibility, we need to consider a system which, unlike the present one, preserves no SUSY and hopefully provides intrinsically a large effective magnetic field.
- Such an effort is now underway and hopefully we can report the progress soon.

THANK YOU!