A Black Hole as A Particle

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Princeton University

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What is a Black Hole?

The Black Hole I understand is:

-1.40135791465086

-1.40135791465086	-1.40135791465080	-1.39627342166226
-1.39627342166226	-1.39341929943020.	-1.39341929943018
-1.39094803659094	-1.39094803659091	-1.38882328162496
-1.38882328162486	-1.38648636076752	-1.38648636076747
-1.38417212341754	-1.38417212341751	-1.38197461694833
-1.38197461694832	-1.38001241876809	-1.38001241876807
-1.37818654140445	-1.37818654140443	-1.37616748049896
-1.37616748049893	-1.37348497184779	-1.37348497184774

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-1.38417212341754	-1.38417212341751	-1.38197461694833
-1.38197461694832	-1.38001241876809	-1.38001241876807
-1.37818654140445	-1.37818654140443	-1.37616748049896
-1.37616748049893	-1.37348497184779	-1.37348497184774

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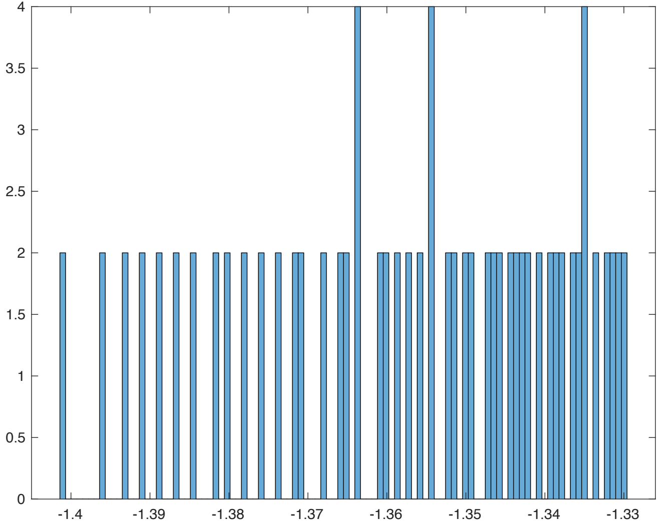
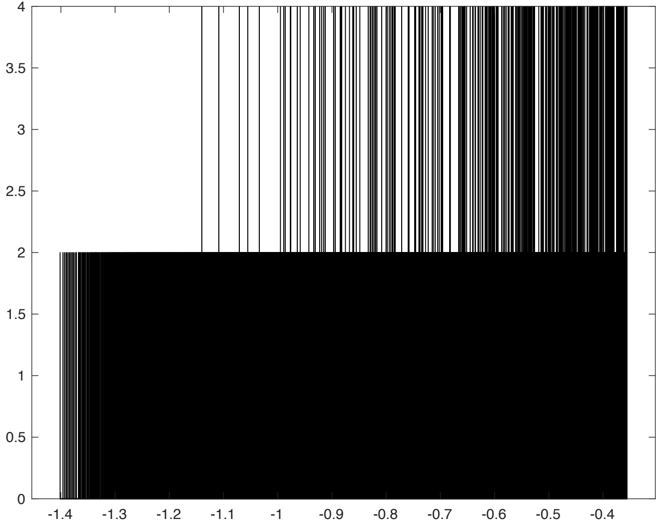
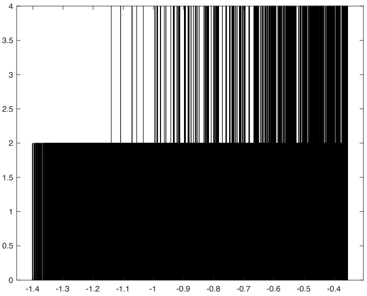
This is what a Black Hole looks like to me, it's complicated and random.

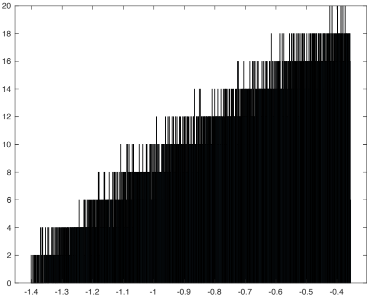


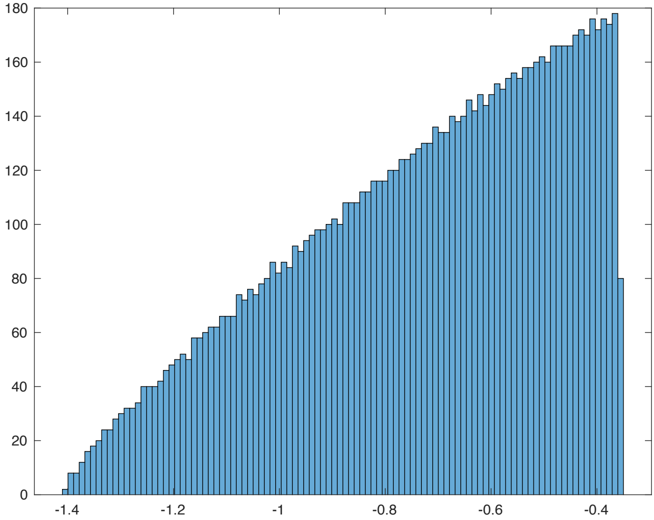
4
3.5
3
2.5
2
1.5
1
0.5
0
-1.4
-1.39
-1.38
-1.37
-1.36
-1.35
-1.34
-1.33

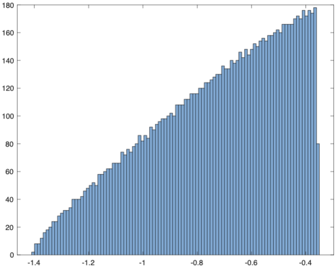

4
3.5
3
2.5
2
1.5
1
0.5
0
-1.4
-1.3
-1.2
-1.1
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4

We can do a little bit coarse grain.






180
160
140
120
100
80
60
40
20
0
-1.4
-1.2
-1
-0.8
-0.6
-0.4



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This is a Near-Extremal Black Hole.

-1.40135791465086	-1.40135791465080	-1.39627342166226
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This is a Near-Extremal Black Hole. This is the beauty of Gravity.


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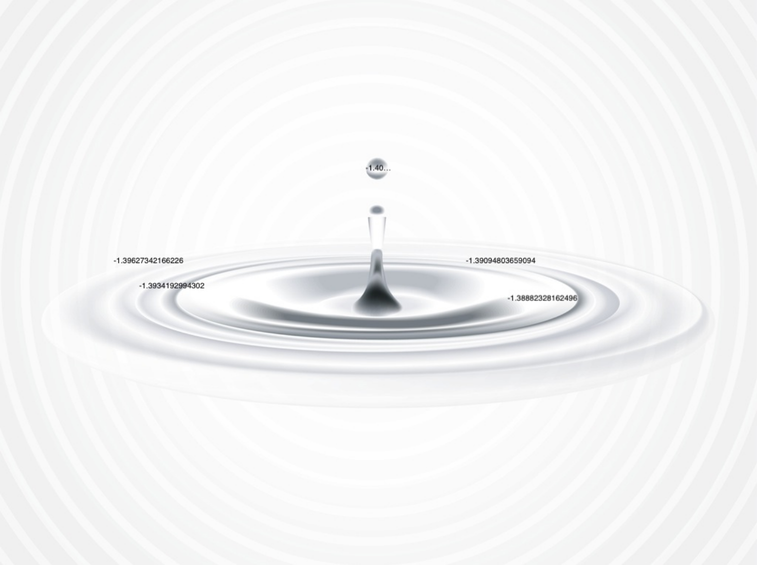
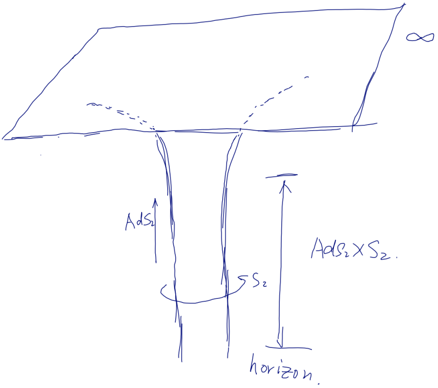

-1.40...
-1.39627342166226
-1.39094803659094
-1.3934192994302
-1.38882328162496

I will ignore:

Near-Extremal black holes have a universal structure near their horizons: there is an AdS_2 throat with a slowly varying internal space. [See Finn's talk in the morning]



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$$I = \underbrace{-\frac{\phi_0}{2}\left(\int R + 2\int_{\partial_M} K\right)}_{\text{Einstein-Hilbert Action}} \underbrace{-\frac{1}{2}\left(\int_M \phi(R+2) + 2\int_{\partial M} \phi_b K\right)}_{\text{Jackiw-Teitelboim action}}. \tag{1}$$

where the dilaton field $\phi + \phi_0$ represents the size of internal space.

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where the dilaton field $\phi + \phi_0$ represents the size of internal space. We have separated the size of internal space into two parts: ϕ_0 is its value at extremality. It sets the value of the extremal entropy which comes from the first term in (1). ϕ is the deviaton from this value.

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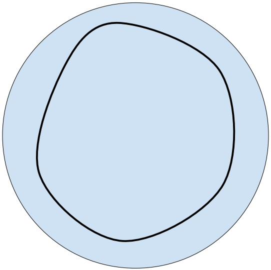
We will mainly study the Euclidean property of the Black Hole, which is corresponding to put this system on a disk:



The boundary is the cut of the Euclidean space where the size of S_2 (the dilaton field) has a relative order one amount of change.

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Such a problem in flat space was considered by Polyakov where he shows that the following problem is directly related to a nonrelativistic particle propagator:

$$\int \mathcal{D}\vec{x} e^{-m_0 \tilde{\tau}} \delta(\dot{\vec{x}}^2 - 1)$$

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μ^2 is the regularized mass and $\tilde{\tau}$ is related to τ by a multiplicative renormalization.

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$$S = \int du \frac{\dot{x}^2 + \dot{y}^2}{y^2} + ib \int du \frac{\dot{x}}{y} - (b^2 + \frac{1}{4}) \int du \ , \qquad b = iq \quad (6)$$

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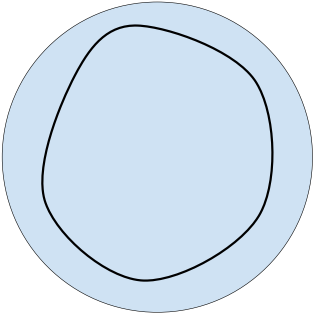
If b is real we will call it a magnetic field, when q is real we will call it an "electric" field.

We see a close connection between the 2d gravity problem and a particle quantum mechanics.



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However there is a sweet limit that avoids all those issues. That is the large q limit. Basically when q is large, it pushes the boundary particle to the asymptotic infinity and demands that the extrinsic curvature to be close to 1. Therefore there will be no self-intersecting curves and the contribution of matter field will be local and only affects the overall coefficient as demanded by symmetry.

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This result was first obtained by [Stanford-Witten] and later recovered by [Bagrets-Altland-Kamenev],[Mertens-Turiaci-Verlinde] and [Kitaev-Suh] by relating this limit to the Schwarzian action.

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The ordering is from the θ function, and we mod out a SL(2,R) group because that is a redundancy in our description.

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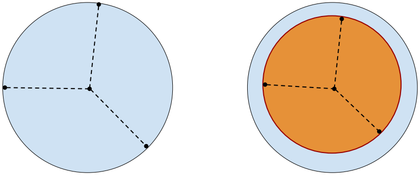
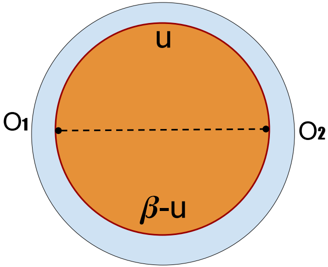


For example, Let us consider the case of two point function, where we have the gravitational feynman diagram as follows:

$$\langle O_1(u)O_2(0)\rangle_{QG} = \tag{14}$$



Written in terms of formula, we have the QFT two point function:

$$\langle O_1(\boldsymbol{x_1})O_2(\boldsymbol{x_2})\rangle_{\mathrm{QFT}} = z_1^{\Delta} z_2^{\Delta} \frac{1}{|x_1 - x_2|^{2\Delta}}. \tag{15}$$

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$$\frac{1}{\mathrm{V(SL(2,R))}} \int_{x_1 > x_2} \frac{dx_1 dx_2 dz_1 dz_2}{z_1^2 z_2^2} \int_0^{\infty} ds_1 ds_2 \rho(s_1)\rho(s_2) e^{-\frac{s_1^2}{2}u - \frac{s_2^2}{2}(\beta - u)}$$
$$\times K_{2is_1}\left(\frac{4\sqrt{z_1 z_2}}{|x_1 - x_2|}\right) K_{2is_2}\left(\frac{4\sqrt{z_1 z_2}}{|x_1 - x_2|}\right)\left(\frac{\sqrt{z_1 z_2}}{|x_1 - x_2|}\right)^{2\Delta+2}. \tag{16}$$

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Where I have already put in the explicit formula for the gravitational propagator. This integral can be done first use SL(2,R) to gauge fix $z_1 = z_2 = 1$ and $x_2 = 0$ and then we can do the spatial integral.

The final result is the following:

$$\begin{aligned}\langle O_1(u)O_2(0)\rangle_{QG} &= \frac{1}{\mathcal{N}}\int ds_1 ds_2 \rho(s_1)\rho(s_2) e^{-\frac{s_1^2}{2}u-\frac{s_2^2}{2}(\beta-u)} \\ &\times \frac{|\Gamma(\Delta - i(s_1+s_2))\Gamma(\Delta + i(s_1-s_2))|^2}{2^{2\Delta+1}\Gamma(2\Delta)}\end{aligned} \tag{17}$$

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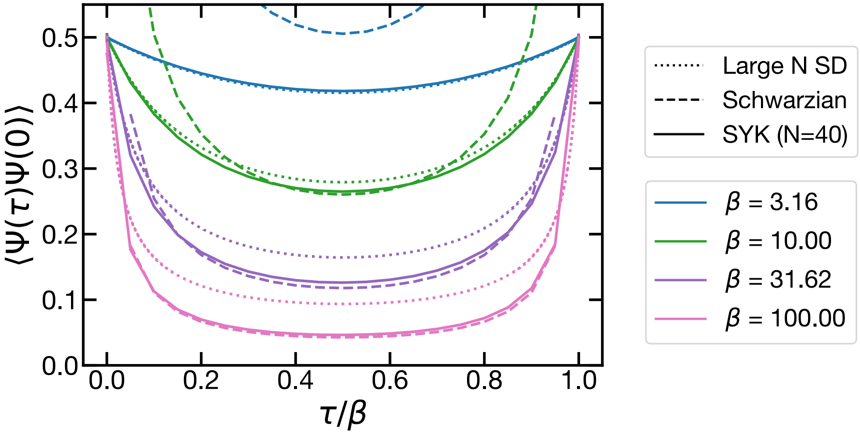
The same result was obtained by [Bagrets-Altland-Kamenev] and [Mertens-Turiaci-Verlinde] using Liouville theory approach.
This exact two point function can be directly compared with exact diagonalization of SYK models which at low energy have a holographic dual of AdS_2.



[Kobrin-Yang-Yao-et al] (To be published)

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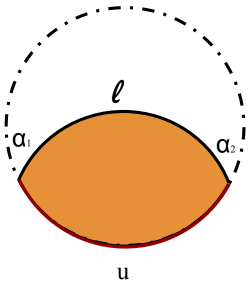

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$$\boxed{\Psi(u;\ell) = \frac{2}{\pi^2 \ell}\int_0^\infty ds s \sinh(2\pi s) e^{-\frac{s^2}{2}u} K_{2is}(\frac{4}{\ell}), \qquad \ell = \frac{|x_1 - x_2|}{\sqrt{z_1 z_2}}.} \tag{18}$$

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whose saddle point equation matches with the classical evaluation of the WdW wavefunction in [Harlow-Jafferis].

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The length of the Einstein-Rosen Bridge has linear growth classically was conjectured to relate with the complexity growth of the system, our result shows that the geometry maintains this behavior at the highly quantum limit. Actually this was first predicted by Susskind in paper [Black Holes and Complexity Classes].

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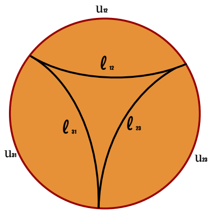

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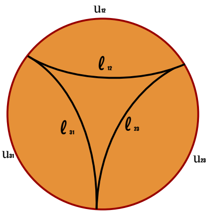

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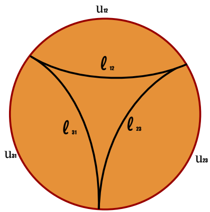

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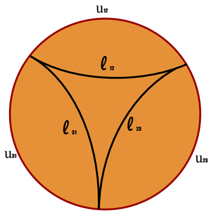

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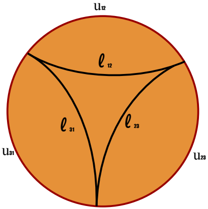

That is there are three wavefunctions glued together with the interior. The path integral of the interior consists of product of three wilson lines. Let's call this product $I(\ell_{12}, \ell_{23}, \ell_{31})$, it satisfies:

$$I(\ell_{12}, \ell_{23}, \ell_{31}) = \frac{16}{\pi^2} \int_0^\infty d\tau \tau \sinh(2\pi\tau) K_{2i\tau}(\frac{4}{\ell_{12}}) K_{2i\tau}(\frac{4}{\ell_{23}}) K_{2i\tau}(\frac{4}{\ell_{31}}) \tag{21}$$

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If we understand this partition function from the point of view of inner product of two states, then we can understand this interior as a gravitational scattering amplitude.
This property is very useful for calculating higher point functions in our previous formula.

Future directions:
Understand the finite q theory [Kitaev-Suh] with proper quantization (Polymer).
Check the quantum gravity effect in other holographic models in Near-Extremal Background. [Larsen], [Papadimitriou]...
Effects on RG flow from gravitational backreaction.


谢谢！